



# FOSSEE Summer Fellowship Report

On

Mathematical Applications Project using GeoGebra

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September 11, 2023

# Acknowledgment

I would like to express my immense gratitude to the FOSSEE Team at IIT Bombay for allowing me to be a part of this fellowship. It has been a great learning journey. Creating GeoGebra projects to give a visual aid for geometric topics in higher level mathematics is essential to provide better intuitive understanding. This internship has given me the possibility to gain knowledge and experience in triangle centers, circle theorems, packing problems, and other geometric constructions.

I would like to thank Ms. Madhuri Ganapathi for her constant guidance and support. Her valuable suggestions helped me improve my work. She showed exemplary patience while guiding me through this endeavour. I would also like to express my profound gratitude to Prof. Kannan M Moudgalya for providing me with this opportunity.

This fellowship is a major milestone in my professional development. Going ahead, I will try to adapt these skills in my mathematical research. I hope to continue working with FOSSEE and offer more GeoGebra Projects into the Open Access World.

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# Chapter 1

## Overview

Spoken Tutorial and Soul project of FOSSEE (Free/Libre and Open Source Software for Education) are projects at Indian Institute of Technology Bombay. They aim to promote the use of open source tools throughout India and reduce the use of proprietary software.

During my three-month fellowship, I was asked to create projects using GeoGebra for Mathematics. To create these projects, I did extensive research, created the constructions on Calculus and Statistics in GeoGebra.

I provide below a brief idea of each of my projects, describing in brief, the topic, and the construction procedure for each.

## Chapter 2

# FOSSEE and SOUL

### 2.1 FOSSEE

FOSSEE(Free/Libre and Open Source Software for Education) project promotes the use of FLOSS tools in academia and research. The FOSSEE project is part of the National Mission on Education through Information and Communication Technology (ICT), Ministry of Education (MoE), Government of India. Science Open source Software for Teaching Learning or SOUL for short is one of the projects which is promoted by FOSSEE.

### 2.2 SOUL

SOUL (Science OpensoUrce Software for Teaching Learning) is an attempt to put together popular ICT software used as teaching/learning tools by the community of educators and learners in basic concepts and advanced learning of Science subjects. These software can be used as ICT tools for topics in science subjects in classroom teaching and learning.

Learners can learn the software using the resources available for each of these software on SOUL website. They can create a project on a particular topic using the available software.

This website can be used as a platform to showcase the completed projects using the software promoted on this page. These projects will be available as resource material for all the interested educators and learners.

# Chapter 3

## GeoGebra Constructions in Calculus

I worked on the following projects. A short description, steps, and a link to the exported web pages of each of the projects is provided below:

### 3.1 Trigonometric Limits

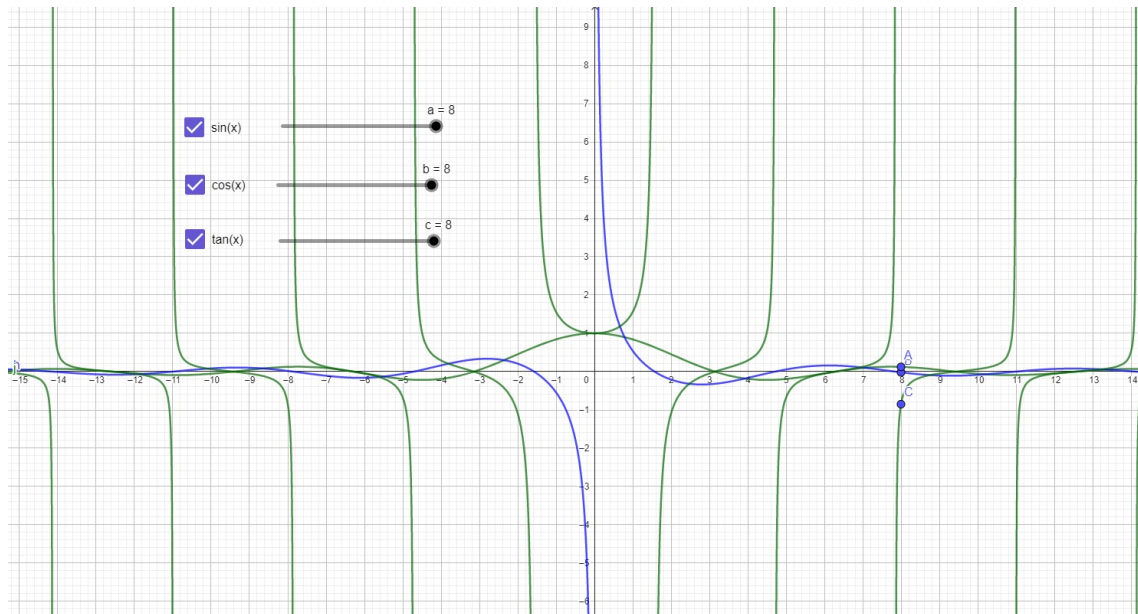


Fig: 3.1: Trigonometric Limits for sine, cosine, and tan

Limits help us understand how functions behave as their input values approach specific points. In the case of trigonometric functions, limits allow us to determine what happens to the function as the input gets closer and closer to a particular value.

For example, consider the function  $f(x) = \sin(x)/x$ . As  $x$  approaches 0, we want to know what happens to the function. By calculating the limit of  $f(x)$  as  $x$  approaches 0 (written as  $\lim_{x \rightarrow 0} f(x)$ ), we can find that the limit is equal to 1. This means

that as  $x$  gets closer and closer to 0, the value of  $f(x)$  tends to 1. Limits provide insights into the behaviour of trigonometric functions near specific points, which is useful in various fields such as calculus, physics, and engineering.

### Steps

1. Open GeoGebra 6 and Create a new file.
2. Add a trigonometric function.  
In the Algebra view, locate the input box at the top. Type the trigonometric function " $f(x) = \sin(x)/x$ ," to represent the limit of  $\sin(x)/x$  as  $x$  approaches 0.
3. Include a slider.  
To demonstrate the concept of a limit, add a slider to control the value of  $x$ . In the Algebra view, click on the "+" icon and select "Slider." Specify the name as " $a$ ", minimum as "-8", maximum as "8", and increment as "1" for the slider.
4. Define the limit function.  
Create a new function that represents the limit of the trigonometric function. In the Algebra view, click on the "+" icon and select "Function." Use the "Limit" command to define the limit function. Now create a function " $g(a) = \text{Limit}(f(x), a)$ " for the limit of  $\sin(x)/x$  as  $x$  approaches  $a$ .
5. Visualize the limit  
Select the "Point" tool from the toolbar. Place the point on the graph to represent the limit value. Use the input bar at the bottom to enter the coordinates of the point as " $(a, g(a))$ ".
6. Type the trigonometric function " $h(x) = \cos(x)/x$ ," to represent the limit of  $\cos(x)/x$  as  $x$  approaches 0. and follow the same steps.
7. Type trigonometric function " $j(x) = \tan(x)/x$ ," to represent the limit of  $\tan(x)/x$  as  $x$  approaches 0. and follow the same steps.

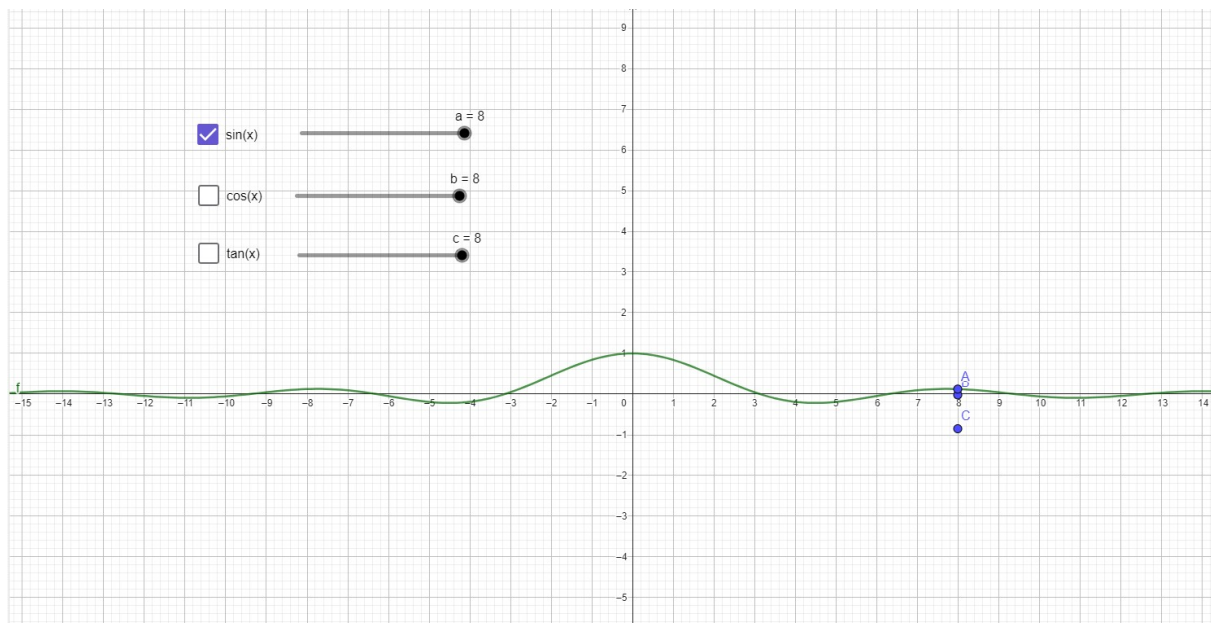


Fig: Trigonometric limits for Sin

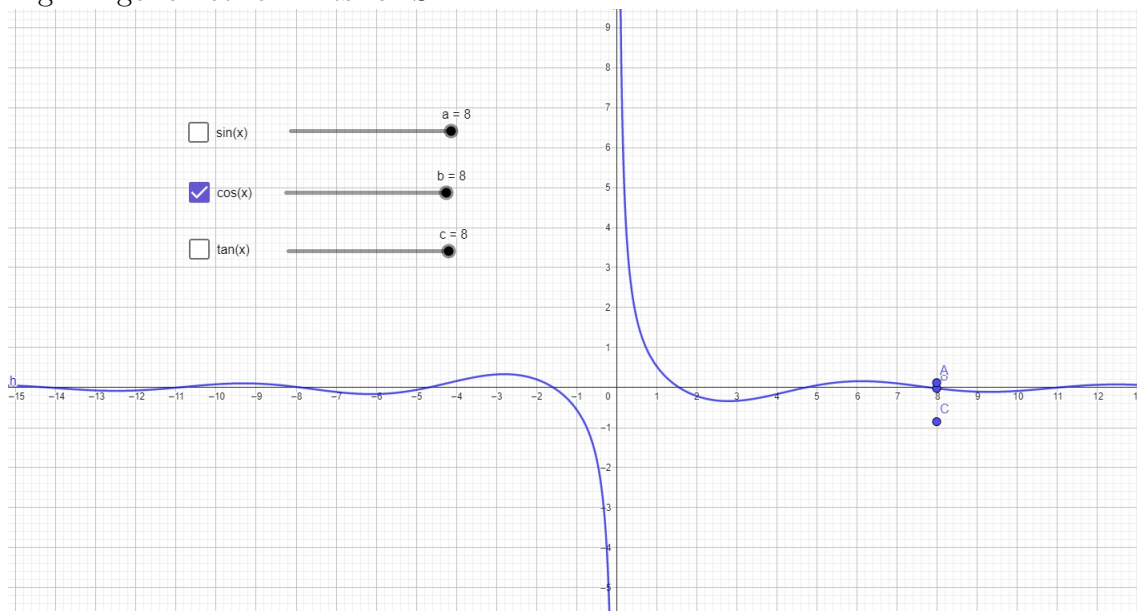


Fig: Trigonometric limits for Cos



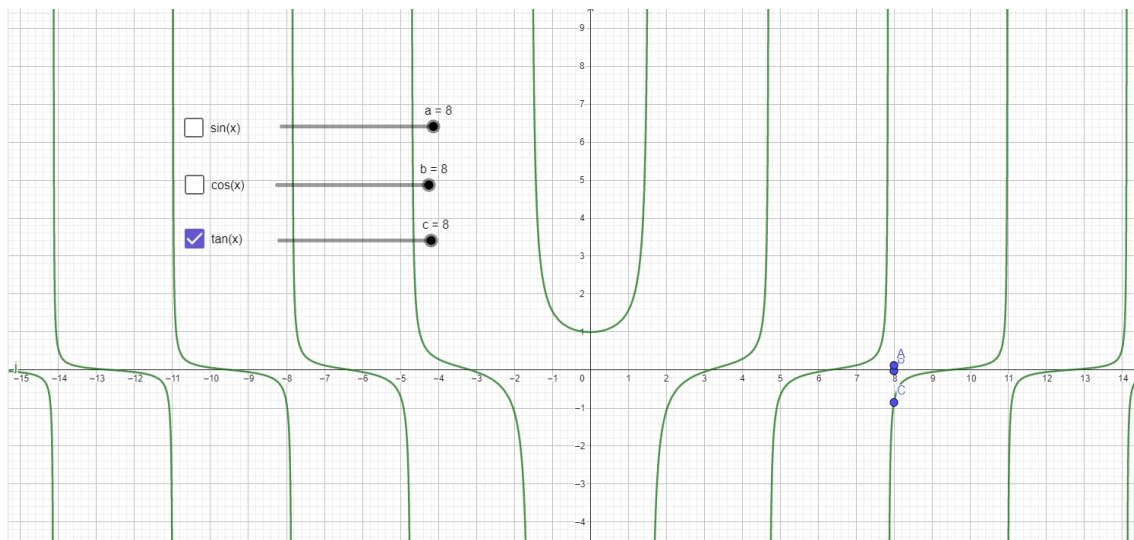


Fig: Trigonometric limits for Tan

<https://www.geogebra.org/m/h6ykaby4>

## 3.2 L'Hopital's Rule

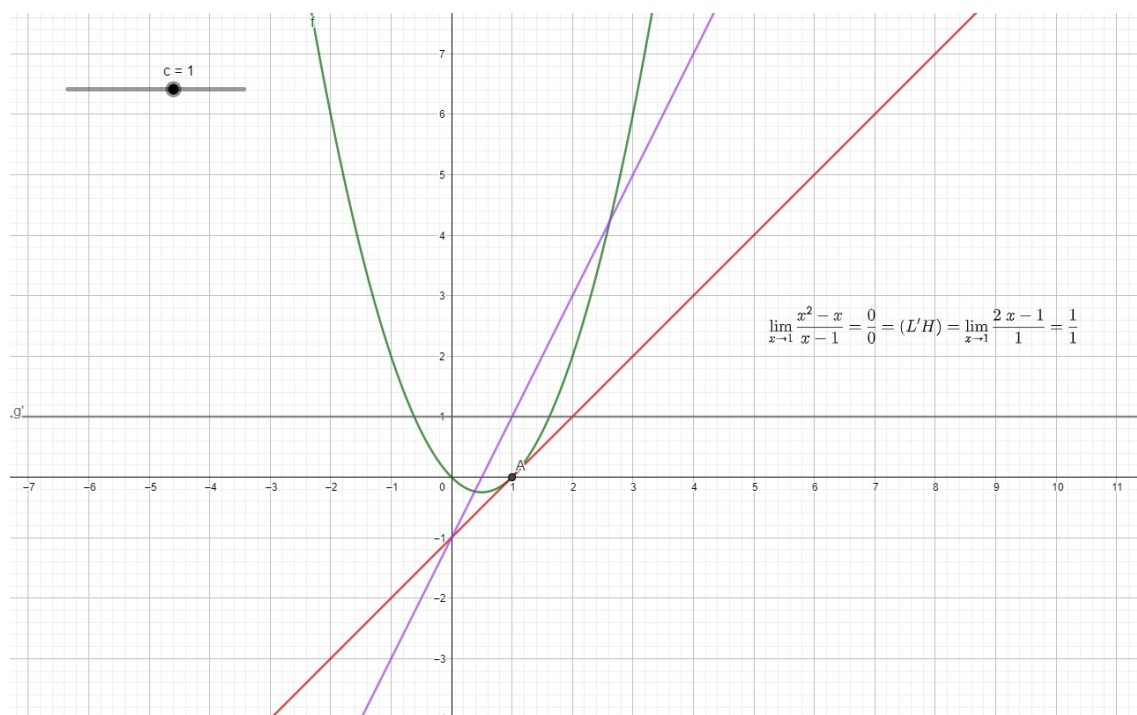


Fig: Implementation of L'Hopital's Rule

Here I have used GeoGebra to explore the application of L'Hopital's Rule in evaluating limits. Two functions,  $f(x)$  and  $g(x)$ , were defined and their derivatives determined. A slider "c" symbolizing the approach value was integrated, allowing

dynamic analysis. The values of  $f(c)$ ,  $g(c)$ ,  $f'(c)$ , and  $g'(c)$  were computed to establish a base for the calculations. By plotting a point A at  $(c, 0)$  and presenting a text box with the expression  $\lim_{x \rightarrow c} \frac{f}{g} = \frac{a}{b} = (L'H) = \lim_{x \rightarrow c} \frac{f'}{g'} = \frac{d}{e}$ , the interaction between the slider and visual elements highlighted the significance of L'Hopital's Rule in resolving limits. This hands-on approach fostered a deeper comprehension of the mathematical concept.

### Steps

1. Open GeoGebra and create a new file.
2. In the Algebra View, define the functions  $f(x)$  and  $g(x)$  as  $f(x) = x^2 - x$  and  $g(x) = x - 1$ , respectively. Also, find the derivatives of  $f(x)$  and  $g(x)$ , denoted as  $f'(x)$  and  $g'(x)$ , which are  $f'(x) = 2x - 1$  and  $g'(x) = 1$ , respectively.
3. Create a slider named "c" with the value of 1. This slider represents the value that  $x$  approaches as it tends to  $c$ .
4. Calculate the values of  $a$ ,  $b$ ,  $d$ , and  $e$  by evaluating the functions and derivatives at the given value of  $c$ :

Set  $a$  as the result of evaluating  $f(c)$ .

Set  $b$  as the result of evaluating  $g(c)$ .

Set  $d$  as the result of evaluating  $f'(c)$ .

Set  $e$  as the result of evaluating  $g'(c)$ .

5. Create a point A at coordinates  $(c, 0)$  to represent the point on the graph.
6. Add a text box to display the L'Hopital's rule and the limit expression. Write the expression " $\lim_{x \rightarrow c} \frac{f}{g} = \frac{a}{b} = (L'H) = \lim_{x \rightarrow c} \frac{f'}{g'} = \frac{d}{e}$ " to represent the equation.
7. Interact with the graph by moving the slider "c" and observe how the graphs and the text box change accordingly.

<https://www.geogebra.org/m/pyxknyh3>

### 3.3 Midpoint, Trapezoidal, Simpson's Rules

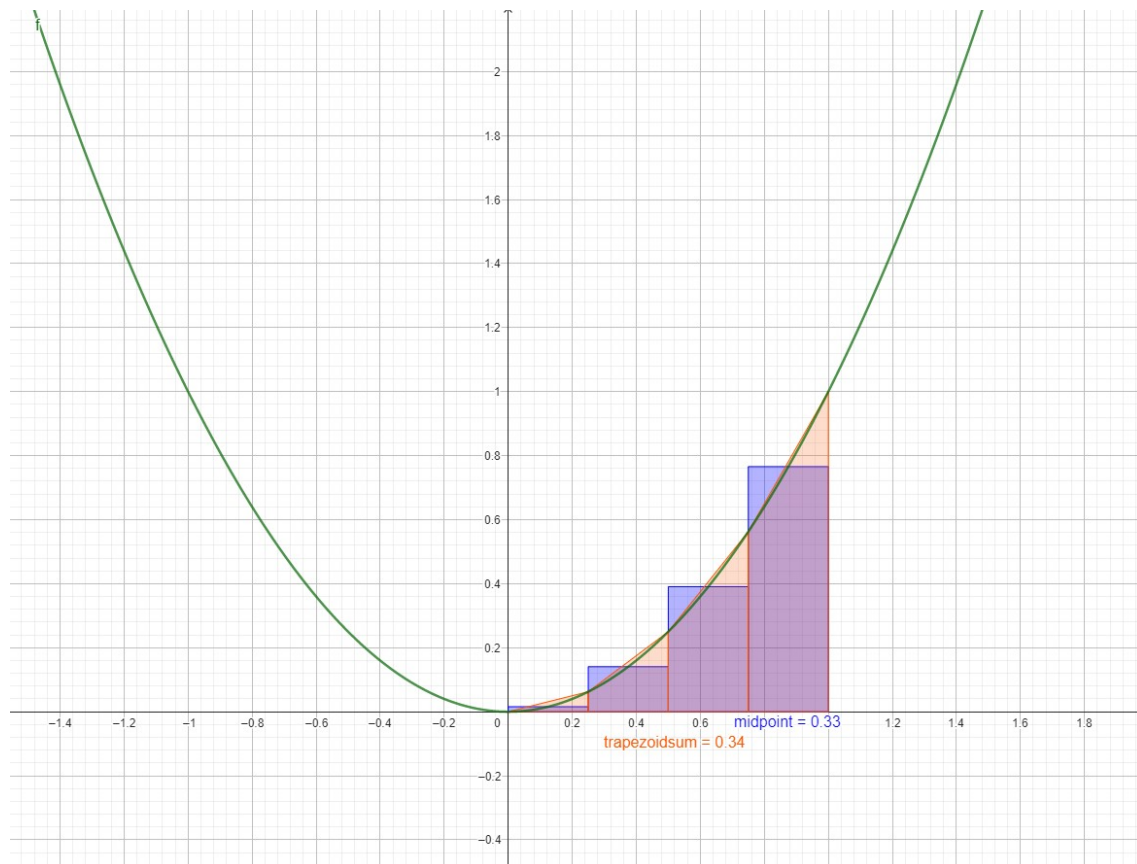


Fig: Midpoint, Trapezoidal, Simpson's Rules

In this project, I used numerical integration techniques. The process involved defining a quadratic function,  $f(x)$ , and subsequently applying integration techniques to approximate definite integrals. The midpoint rule was employed using the RectangleSum function, precisely configuring parameters such as subintervals and displacement.

The TrapezoidalSum function was then utilized for the trapezoidal rule approximation, enhancing understanding through practical implementation. The synthesis of both methods via Simpson's rule, achieved by blending their results with specific weights, culminated in a comprehensive approximation. By seamlessly integrating theory and application, this project offered a hands-on comprehension of numerical integration principles and their real-world significance.

#### Steps

1. Open GeoGebra
2. Create a new file
3. Define the function  $f(x) = x^2$ .
4. Calculate the midpoint rule using the RectangleSum function: Use the RectangleSum function with the parameters:  $f(x)$ , lower limit (0), upper limit (1),

number of subintervals (4), and displacement (0.5).

5. Calculate the trapezoidal rule using the TrapezoidalSum function: Use the TrapezoidalSum function with the parameters:  $f(x)$ , lower limit (0), upper limit (1), and number of subintervals (4).
6. Combine the midpoint and trapezoidal rule using Simpson's rule: Multiply the midpoint sum by  $(2/3)$  and the trapezoidal sum by  $(1/3)$ . Add the results together to get the Simpson's rule approximation.

<https://www.geogebra.org/m/gkcrbyns>

### 3.4 Integration by Parts

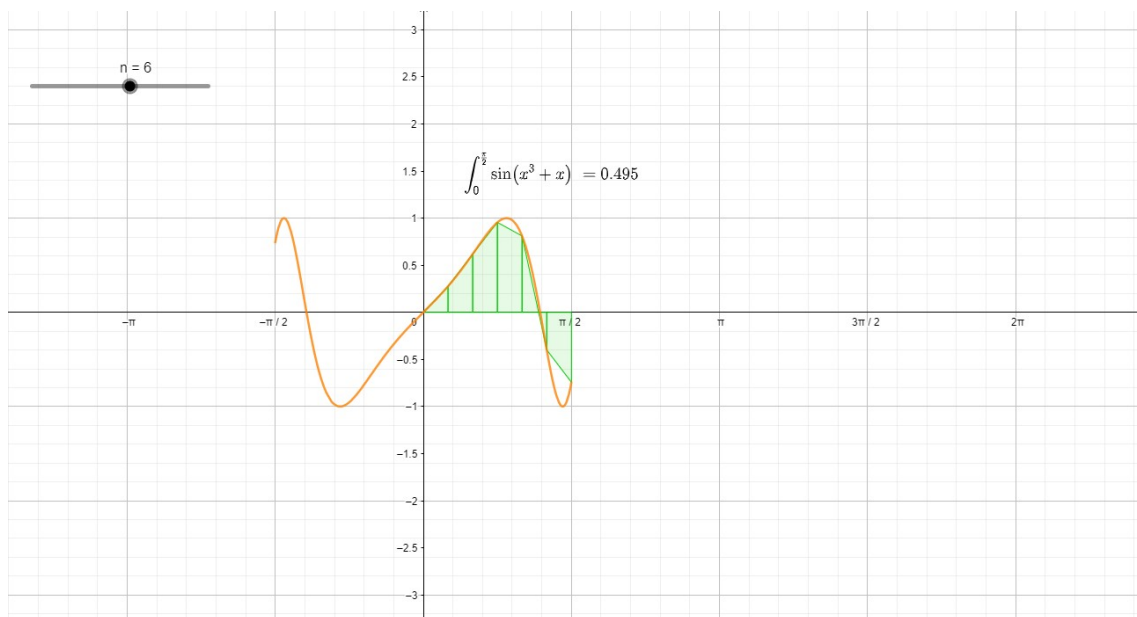


Fig: Integration by Parts

In this GeoGebra project, the utilization of numerical integration techniques was explored through practical implementation. A defined function,  $f(x) = \sin(x^3 + x)$ , was employed as the integrand, and constraints were established using  $g(x)$  to define the

function within specific limits. The Trapezoidal Rule, involving six subintervals, was harnessed to approximate the definite integral of  $f(x)$  over the interval  $[0, \pi/2]$ . Through GeoGebra commands, the calculated approximation was revealed, bolstering understanding of the process. The final touch incorporated LaTeX notation to elegantly portray the integral symbol, integration limits, and the computed approximation. By merging theory and application, this project adeptly showcased the potency of numerical integration in estimating complex functions and unveiled the symbiotic relationship between mathematical concepts and practical execution.

## Steps

1. Open GeoGebra and create a new file.
2. Define the function  $f(x)$  as  $f(x) = \sin(x^3 + x)$ . This function represents the integrand.
3. Define the function  $g(x)$  as  $g(x) = \text{if}((-\pi/2) \leq x \leq (\pi/2), f(x))$ . This function represents the integrand within the given limits of integration.
4. Set the value of  $n$  as 6. This represents the number of subintervals for the Trapezoidal Rule.
5. Calculate the approximation of the definite integral using the Trapezoidal Rule. To do this, use the command  $\text{TrapezoidalSum}(f, 0, \pi/2, n)$ . This will compute the integral of  $f(x)$  from 0 to  $\pi/2$  and assign the approximation to the variable 'a'.
6. Display the result. Use the command  $a$  to show the value of the approximation on the screen.
7. To display the integral symbol and the limits of integration in the output, use LaTeX notation. Write ' $\int_0^{\pi/2} f(x)dx = a$ ' to represent the integral of  $f(x)$  from 0 to  $\pi/2$  along with the computed approximation.

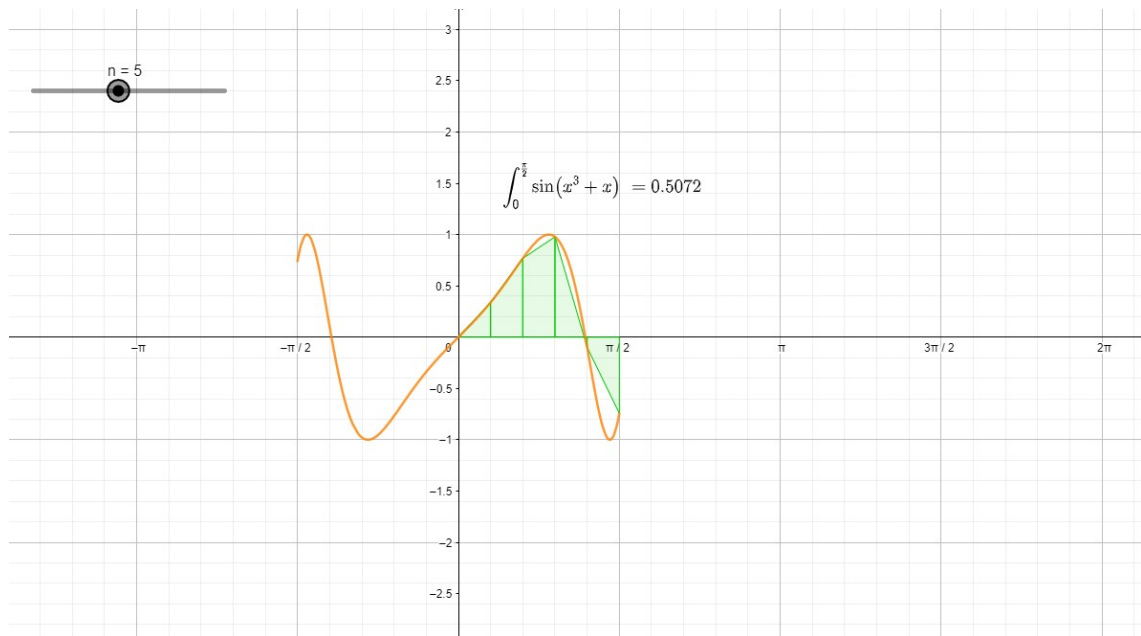


Fig: Integration by parts at  $n = 5$

<https://www.geogebra.org/m/awcj5wnj>

### 3.5 Area between Curve and X-axis(Applications of Integration)

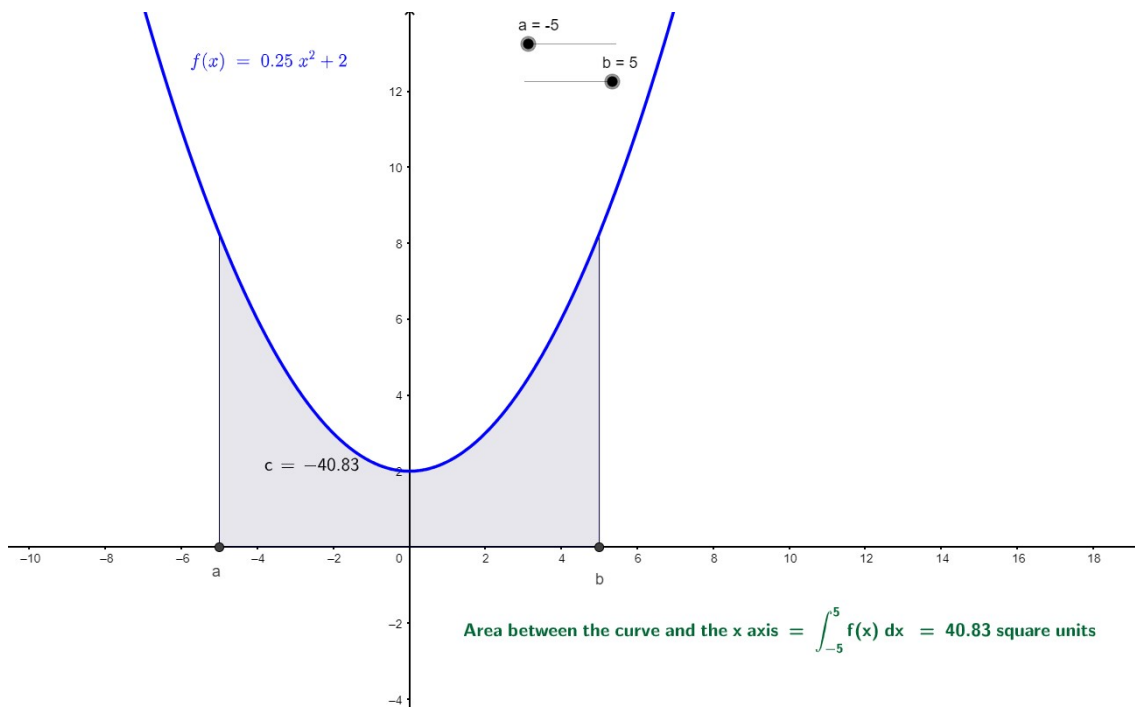


Fig: Area between Curve and X-axis

This GeoGebra project adeptly demonstrates the practical application of integration in calculating the area between a curve and the x-axis. By defining the function  $f(x) = 0.25x^2 + 2$ , the area of interest is identified. Through interactive sliders for limits of integration ( $a$  and  $b$ ), the user can dynamically set the interval. The integration of  $f(x)$  over the interval  $[a, b]$  is calculated and its absolute value obtained to ensure a positive area. By determining the x-intercepts and presenting the formula text, the project visualizes the interval and equation of the area calculation. Through this interactive exploration, the project underscores the integration's utility in determining enclosed areas and provides an illustrative model of mathematical applications in real-world contexts.

#### Steps

1. Define the function:  
 $f(x) = 0.25x^2 + 2$ . This represents the curve for which you want to find the area between it and the x-axis.
2. Create sliders for  $a$  and  $b$ :  
Create sliders for  $a$  and  $b$  to specify the limits of integration. Set the initial values of  $a$  and  $b$  to  $-5$  and  $5$ , respectively.
3. Calculate the integral:  
 $c = \text{Integral}(f, b, a)$ . Calculate the definite integral of the function  $f(x)$  with

respect to x, from  $x = a$  to  $x = b$ . The result is stored in the variable c.

4. Calculate the absolute value of the integral:  
 $g = \text{abs}(c)$ . Take the absolute value of the calculated integral c to ensure a positive value for the area.
5. Find the x-intercepts of the curve:  
 $A = \text{Intersect}(d, x - \text{axis})$  and  $B = \text{Intersect}(e, x - \text{axis})$ . Calculate the x-coordinates where the curve intersects the x-axis. These points will help visualize the interval over which the area is being calculated.
6. Display the area and formula:  
 Area between the curve and the x-axis = integral from a to b of  $f(x) \, dx = g$  square units. Display the formula for the area, including the integral notation and the value of g, in the GeoGebra app.
7. Display the formula text:  
 $\text{text2} = \text{FormulaText}(c, \text{true}, \text{true})$ . Convert the calculated integral c into a formatted formula text and store it in the variable text2. The "true, true" parameters ensure that the text includes the integral symbol and the integration limits.

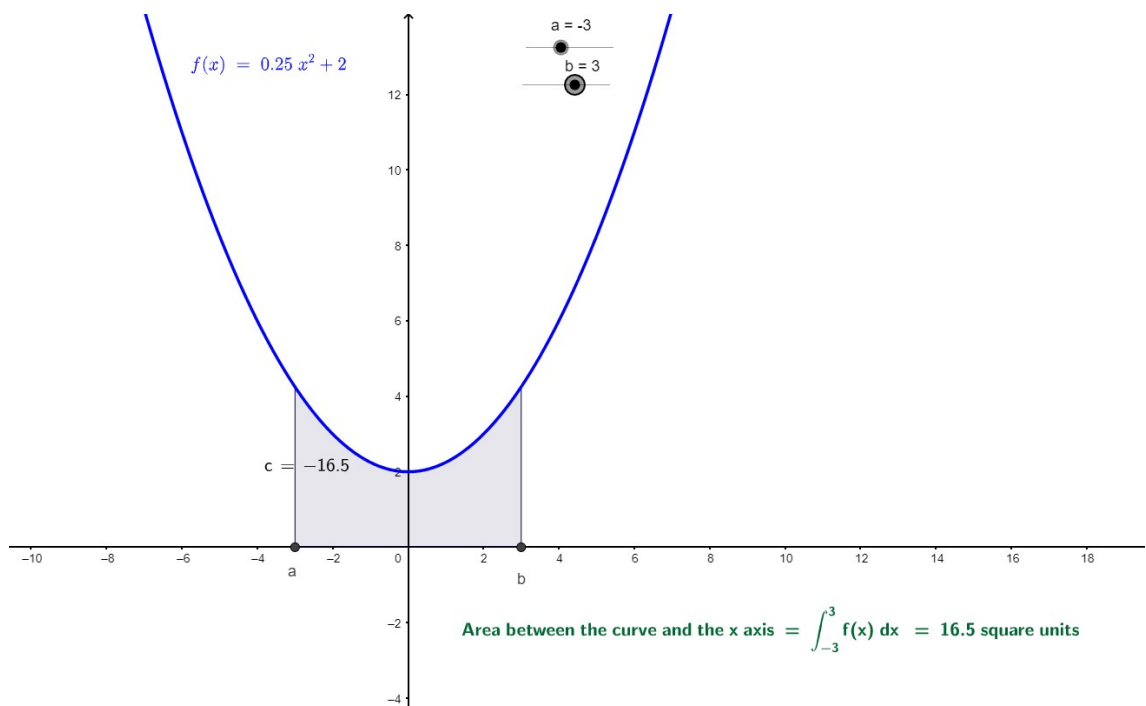


Fig: Area between Curve and X-axis at limits (-3,3)

<https://www.geogebra.org/m/ach6bs7f>

## 3.6 Rolle's Theorem

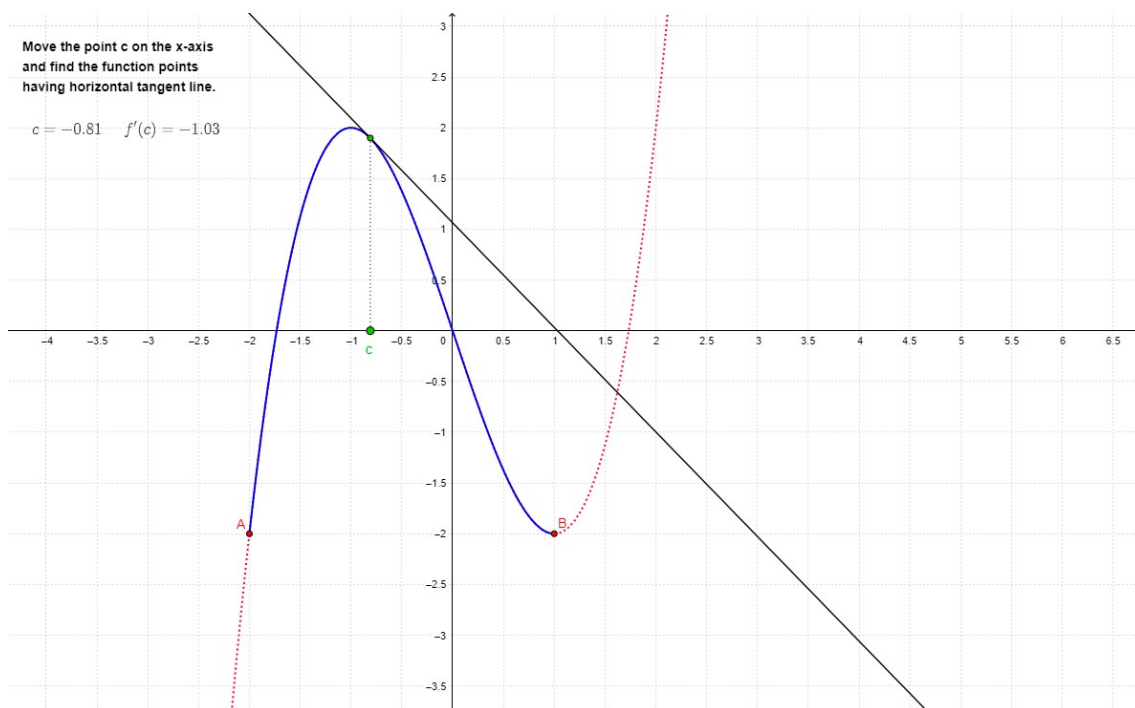


Fig: Rolle's Theorem

This GeoGebra project explores the characteristics of a cubic function and its tangent lines within a defined interval. By defining functions and values, it demonstrates the dynamic nature of calculus concepts. The project showcases tangent lines, intersections, and derivatives, effectively illustrating the relationship between the original function and its derivative. Through interactive manipulation of points on the graph, the project visually underscores the horizontal tangent points where the derivative equals a constant value. This hands-on approach offers an insightful understanding of the interplay between a function and its derivative, demonstrating the power of GeoGebra as a tool for visualizing and comprehending calculus principles in action.

### Steps

1. Open GeoGebra and create a new graphing window.
2. Define the function  $f(x)$  by typing " $f(x) = x^3 - 3x$ ",  $f'(x) = f'(x)$ " in the input bar and press Enter.
3. Define the function  $p(x)$  using the If statement to restrict its domain to the interval  $[a, b]$ . Type " $p(x) = If(a \leq x \leq b, f(x))$ " in the input bar and press Enter.
4. Create a tangent line to the graph of  $f(x)$  at point D. Type " $c1 = Tangent(D, f)$ " in the input bar and press Enter.
5. Define the values for a and b by typing " $a = -2$ " and " $b = 1$ " in the input bar, respectively, and pressing Enter.



6. Calculate the derivative of  $f(x)$  at point C. Type " $e = f'(x(C))$ " in the input bar and press Enter.
7. Define the coordinates for points A and B. Type " $A = (a, f(a))$ " and " $B = (b, f(b))$ " in the input bar and press Enter.
8. Create point C as the intersection of the x-axis and the segment AB. Type " $C = \text{Point}(\text{Segment}((x(A), 0), (x(B), 0)))$ " in the input bar and press Enter.
9. Create point D as the intersection of the tangent line and the x-axis. Type " $D = \text{Intersect}(c1, x\text{-axis})$ " in the input bar and press Enter.
10. Create a segment from point D to point C. Type " $d = \text{Segment}(D, C)$ " in the input bar and press Enter.
11. Add a text label to display the values of  $x(C)$  and  $f'(c)$ . Type " $c = x(C)\text{Text}('c = ' + \text{ToString}(c) + 'f'(c) = ' + \text{ToString}(e), -2, f(a) + 10)$ " in the input bar and press Enter.
12. Move point C along the x-axis by clicking and dragging it. As you move the point, the text label will update to show the corresponding values of  $x(C)$  and  $f'(c)$ .
13. Observe the function points where the tangent line is horizontal. These points will correspond to the values of  $x(C)$  where  $f'(c)$  equals the constant value  $e$ .

<https://www.geogebra.org/m/uxrhmwjd>

### 3.7 Mean Value Theorem

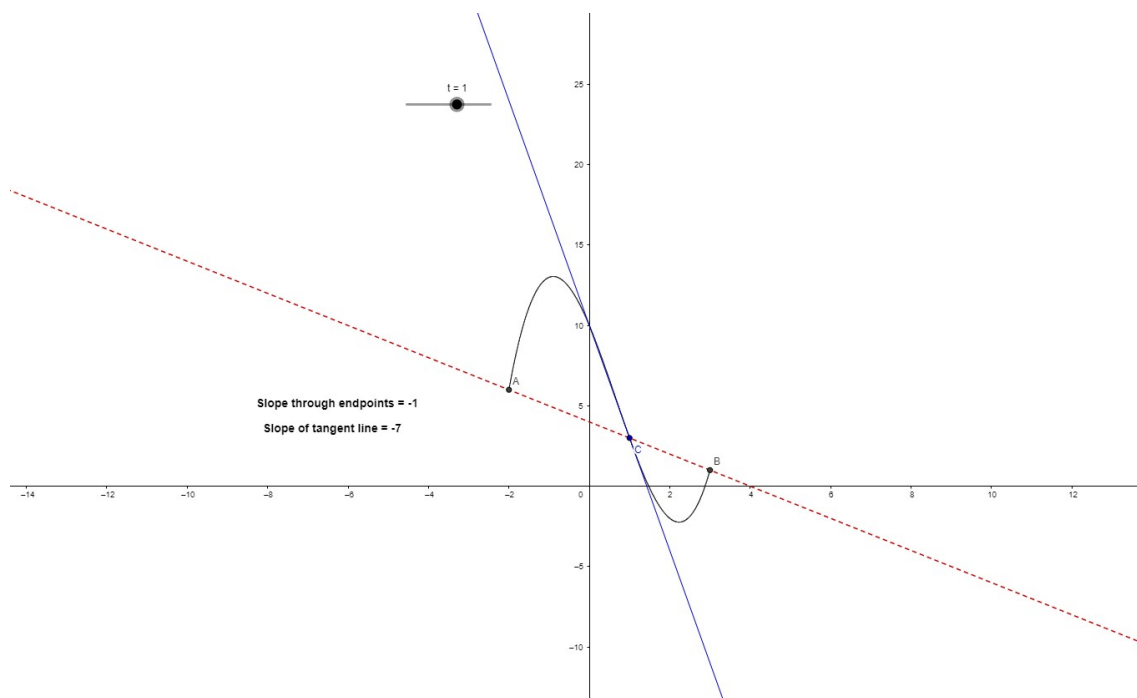


Fig: Mean Value Theorem at  $t=1$

This GeoGebra project delves into the Mean Value Theorem's practical application, unraveling its significance in the context of a defined cubic function. By employing graphical and analytical tools, the project elucidates the theorem's conditions. Through interactive point plotting and slider manipulation, the relationship between tangent line slopes and secant line slopes is scrutinized. The project effectively visualizes the theorem's essence, illustrating how a parallel tangent and secant line indicates a point within the interval satisfying the theorem's criteria. This hands-on exploration fosters a comprehensive understanding of calculus concepts and their tangible implications, highlighting GeoGebra's capability in bridging theory with real-world observations.

### Steps

1. Open GeoGebra and create a new file.
2. Define the function  $f(x)$  using the input provided:  $f(x) = x^3 - 2x^2 - 6x + 10$ . To do this, click on the "Function" icon in the toolbar and enter the function in the input box that appears.
3. Plot the graph of the function  $f(x)$  over the interval  $[-2, 3]$ . To do this, click on the "Function" icon again and select "Function" from the drop-down menu. Enter the function  $f(x)$  and the interval  $[-2, 3]$  in the input box.
4. Plot the point A at  $(-2, f(-2))$ . To do this, click on the "Point" icon in the toolbar and click at the coordinates  $(-2, f(-2))$ .
5. Plot the point B at  $(3, f(3))$ . Like step 4, click on the "Point" icon and click at the coordinates  $(3, f(3))$ .
6. Define the slider "t" by clicking on the "Slider" icon in the toolbar and assigning it the value of -1.
7. Plot the point C at  $(t, f(t))$ . To do this, click on the "Point" icon, click on the canvas to create a new point, and use the input box to specify the x-coordinate as "t" and the y-coordinate as "f(t)".
8. Calculate the slope of the line passing through points A and B, denoted as  $m_2$ . To do this, click on the "Line through two points" icon in the toolbar, select point A, and then select point B. The slope of the line will be displayed.
9. Calculate the slope of the tangent line to the curve at point C, denoted as  $m_1$ . To do this, click on the "Tangent" icon in the toolbar, select point C, and the tangent line will be displayed along with its slope.
10. Compare the slopes  $m_1$  and  $m_2$ . If they are equal, it indicates that there exists at least one point in the interval  $[-2, 3]$  where the tangent line is parallel to the line passing through A and B, satisfying the conditions of the Mean Value Theorem.
11. Optionally, you can add labels or annotations to the graph and points for clarity and visual appeal.

12. You can interact with the slider "t" to observe how the slope of the tangent line changes as you move the point C along the curve.

<https://www.geogebra.org/m/ydyjjmgs>

## 3.8 Fundamental Theorem of Calculus

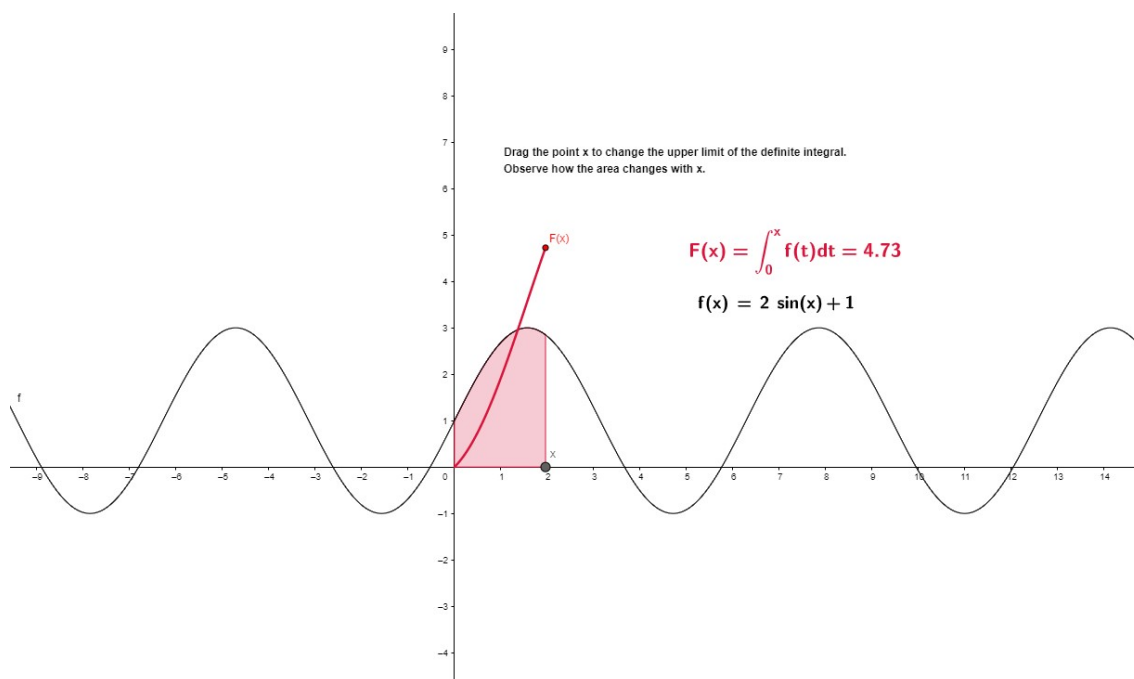


Fig: Fundamental Theorem of Calculus

This GeoGebra project engages with the Fundamental Theorem of Calculus by visually and interactively illustrating its concepts. Through intuitive steps, the definite integral of a given function is explored. By defining functions and variables, the project creates an interactive platform to showcase the relationship between definite integrals and antiderivatives. Points and text elements are strategically placed to aid understanding and facilitate interaction. The project encourages user engagement by allowing the manipulation of the upper limit of the definite integral, providing real-time visualization of area changes. By seamlessly integrating theory and practice, the project offers an insightful demonstration of the Fundamental Theorem's principles and GeoGebra's utility in making complex calculus concepts accessible and comprehensible.

### Steps

1. Open GeoGebra and create a new file.
2. Define the function " $f(x) = 2\sin(x) + 1$ " by typing it into the input bar or using the "Input" tool and selecting "Function."
3. Define the function " $g(x) = If(0 \leq x \leq x(A), Integral(f) + 2)$ " by typing it into the input bar or using the "Input" tool and selecting "Function." This

function calculates the value of the integral of  $f(x)$  and adds 2 if the  $x$ -value is within the range from 0 to  $x(A)$ .

4. Define the variable " $a = \text{Integral}(f, 0, x(A))$ " by typing it into the input bar or using the "Input" tool and selecting "Variable." This variable represents the value of the definite integral of  $f(x)$  from 0 to  $x(A)$ .
5. Define the variable " $c_1 = 0$ " by typing it into the input bar or using the "Input" tool and selecting "Variable." This variable represents the constant lower limit of the definite integral.
6. Create a point "A" on the  $x$ -axis by selecting the "Point" tool and clicking on the  $x$ -axis. This point represents the upper limit of the definite integral.
7. Create a point "B" by selecting the "Point" tool and typing " $(x(A), a)$ " into the input bar. This point represents the coordinates  $(x(A), a)$ , where  $a$  is the value of the definite integral.
8. Create a function " $F(x) = \text{Integral}(f, 0, x)$ " by typing it into the input bar or using the "Input" tool and selecting "Function." This function represents the integral of  $f(x)$  from 0 to  $x$ .
9. Add a text element by selecting the "Text" tool and clicking on the desired location. Type " $F(x) = \int_0^x f(t)dt = a$ " to display the equation of the antiderivative.
10. Add another text element by selecting the "Text" tool and clicking on the desired location. Type "Drag the point  $x$  to change the upper limit of the definite integral. Observe how the area changes with  $x$ " to provide instructions to the user.
11. Add one more text element by selecting the "Text" tool and clicking on the desired location. Type " $\text{text3} = \text{FormulaText}(f, \text{true}, \text{true})$ " to display the formula of  $f(x)$  in the project.
12. Arrange the elements in the construction as desired, adjusting their positions and sizes as needed.
13. We can drag the point "X" to change the upper limit of the definite integral, and observe how the area changes accordingly. The function " $F(x)$ " represents the antiderivative of  $f(x)$ , and the text element "text3" displays the formula of  $f(x)$ .

<https://www.geogebra.org/m/yne4sdr7>

### 3.9 The Intermediate Value Theorem

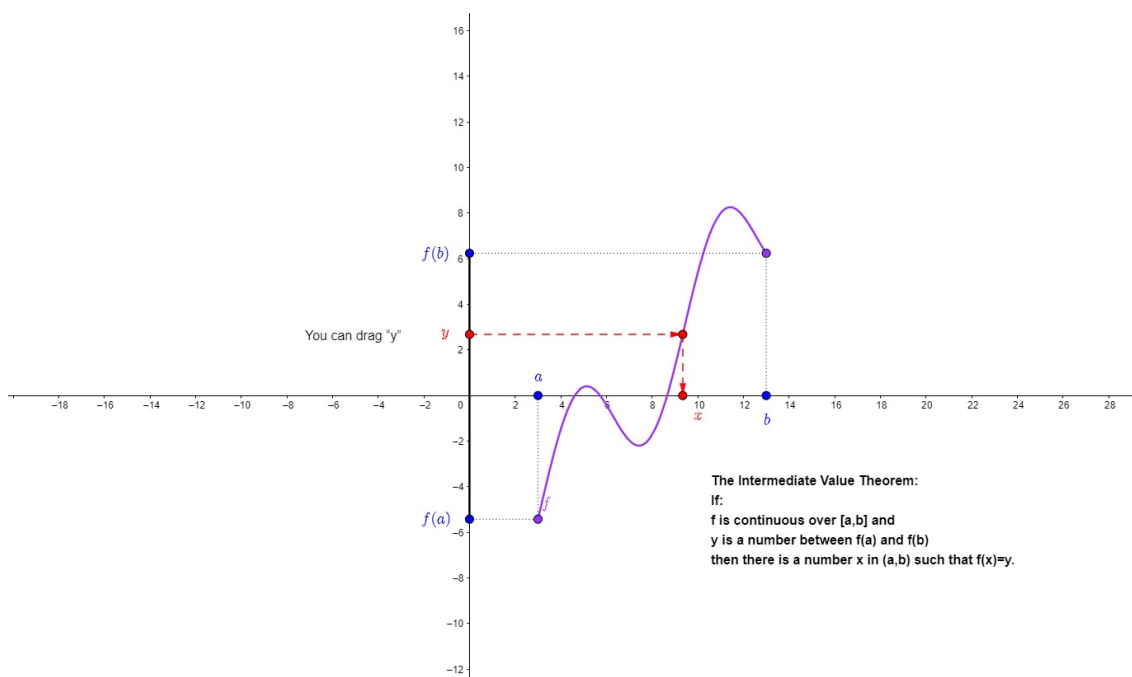


Fig: Intermediate value Theorem

This GeoGebra project offers a dynamic visualization of the Intermediate Value Theorem's principles. Through strategically defined functions, points, and vectors, the project conveys the theorem's concept of continuity within an interval. By creating intersections, lines, and segments, the project vividly illustrates potential x-values that satisfy the theorem's conditions. The use of vectors highlights the directional aspects involved. Additionally, the inclusion of informative text further enhances comprehension, addressing the possibility of multiple solutions. This interactive approach effectively bridges theory and practical application, showcasing GeoGebra's prowess in rendering complex mathematical concepts accessible and tangible. The project provides an engaging exploration of the Intermediate Value Theorem's significance, demonstrating its real-world implications in an intuitive and interactive manner.

#### Steps

1. Open GeoGebra and create a new file.
2. The function  $f(x)$  is defined based on the given conditions, using the If statement to choose between the values of  $g(x)$  when  $x$  is within the interval  $[x(A), x(B)]$ .
3. The line  $l$  is created to be perpendicular to the x-axis at  $x = 0$ . This line will be used to find the intersection point with the graph of  $g(x)$ .
4. The intersection point between the line  $l$  and the graph of  $g(x)$  is calculated using the points A and B. This point represents the possible value(s) of  $x$  that satisfy the Intermediate Value Theorem.

5. Various points are defined with their respective coordinates: A and B represent the given x-coordinates with a y-coordinate of 0. C and D represent the x-coordinates of the intersection point and the corresponding y-values obtained from  $f(x)$ . E and F represent the y-values obtained from  $f(x)$  at  $x(A)$  and  $x(B)$ , with x-coordinates of 0. H represents the first element (index 1) from the list G, which contains the intersectionpoint(s) of  $g(x)$  and the line  $l$ . I represent the x-coordinate of point H with a y-coordinate of 0. Y is a movable point represented by m.
6. The line segments i, j, k, and m are created using the given points to visually represent different segments of the graph.
7. The vector u is calculated based on the points Y and H. This vector represents the direction and magnitude from point Y to the intersection point H.
8. The vector v is determined using the points H and I. This vector represents the direction and magnitude from point H to the x-axis.
9. The text ivt4 is defined to display a message about the possibility of multiple x values satisfying the Intermediate Value Theorem. If there are more than one intersection point in the list G, this note will be shown to indicate that there may be multiple x values satisfying the theorem.

<https://www.geogebra.org/m/syzej44q>

### 3.10 The Squeeze Theorem

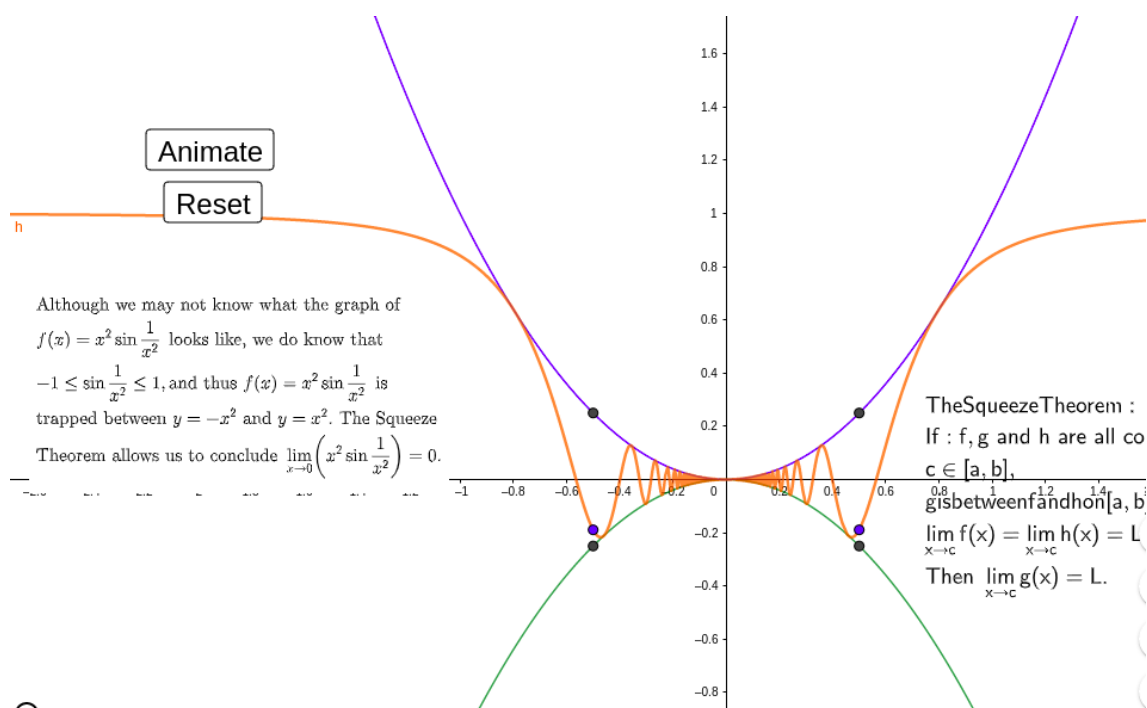


Fig: The Squeeze Theorem

This GeoGebra project elegantly explores the Squeeze Theorem, a fundamental concept in calculus. Through interactive elements, the project vividly demonstrates how functions are bounded and constrained within certain limits. By defining distinct functions and incorporating dynamic sliders, the relationships between the functions are established and visualized. Text explanations effectively introduce the theorem's relevance, while an animation button engages users in observing how the theorem operates in real-time. The inclusion of a reset button ensures flexibility for exploration and repetition. This project merges theory with interactive visualization, offering an insightful understanding of the Squeeze Theorem's applicability and GeoGebra's prowess in conveying complex mathematical principles through an intuitive interface.

## Steps

1. Open GeoGebra and create a new file.
2. Define the functions  $f(x)$ ,  $g(x)$ , and  $h(x)$  based on the given expressions:
 
$$f(x) = -x^2$$

$$g(x) = x^2$$

$$h(x) = x^2 * \sin(1/x^2)$$
3. Set up the sliders  $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$ , and  $i$  to allow for flexible input of numerical values. These sliders will be used to define the coordinates of points in subsequent steps:  
 $a$ ,  $b$ ,  $c$ ,  $d$ ,  $e$ , and  $i$  represent different numerical values.
4. . Define the points  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ , and  $F$  using the given sliders:
 
$$A = (a, f(a))$$

$$B = (b, f(b))$$

$$C = (c, g(c))$$

$$D = (d, g(d))$$

$$E = (e, h(e))$$

$$F = (i, h(i))$$
5. Create the texts  $\text{text1}$  and  $\text{text2}$  to provide explanations and state the Squeeze Theorem:  
 $\text{text1}$  contains an explanation of the function  $f(x)$  and its relationship to the bounds of  $-x^2$  and  $x^2$ . It also concludes that  $f(x) = x^2 * \sin(1/x^2)$  is trapped between  $-x^2$  and  $x^2$ , establishing the application of the Squeeze Theorem. \*  
 $\text{text2}$  presents the statement of the Squeeze Theorem, emphasizing its conditions and explaining that if  $f(x)$  and  $h(x)$  have the same limit  $L$  as  $x$  approaches  $c$ , then  $g(x)$  also approaches  $L$  as  $x$  approaches  $c$ .
6. Add an animate button to the GeoGebra task. This button triggers an animation that demonstrates the behavior of the functions and how the Squeeze Theorem applies. By clicking the animate button, the graph and points will dynamically update according to the changing values of the sliders, showcasing the concept of the Squeeze Theorem in action.

7. Include a reset button in the GeoGebra task. This button allows the user to reset the animation and return the graph to its initial state. Clicking the reset button will restore the original positions of the graph, points, and texts, allowing for a fresh start or repeated demonstrations of the Squeeze Theorem.

<https://www.geogebra.org/m/kweghubh>



# Chapter 4

## GeoGebra Constructions in Statistics

### 4.1 Dot Plot

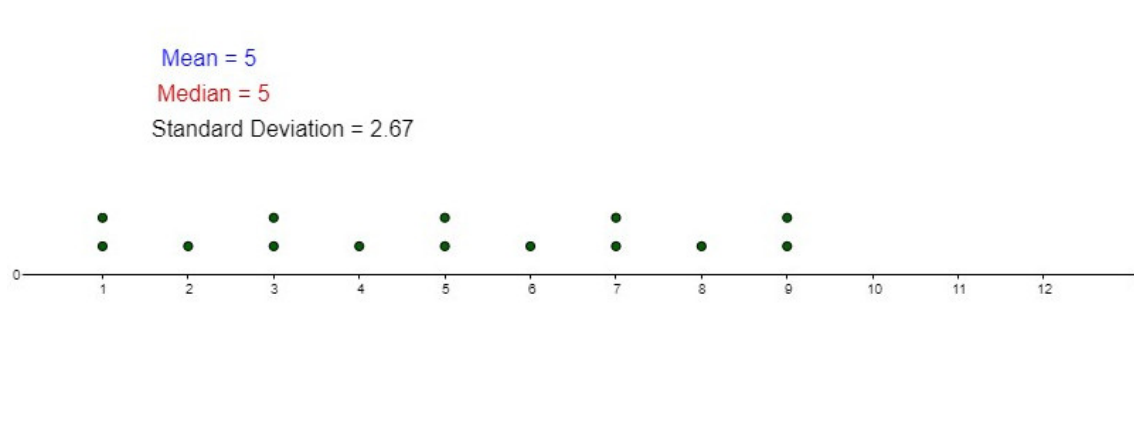


Fig: Dot Plot

	A	B	C	D	E	F	G	H	I	J	K
1	1				Mean	5				4.22	1
2	2				Median:	5					
3	3				St. Dev:	2.67					
4	4										
5	5										
6	6										
7	7										
8	8										
9	9										
10	1										
11	3										
12	5										
13	7										
14	9										
15											
16											
17											
18											
19											
20											

Fig: Coordinates for the Dot Plot

This GeoGebra project introduces a dynamic Dot Plot generator for data visualization. By utilizing spreadsheets and lists, the project effortlessly transforms numerical data into a dot plot format. Through a step-by-step process, the project creates a dot plot on a graph, automatically updating as data is entered. Additionally, the mean, median, and standard deviation are dynamically calculated and displayed on the graph through text objects. The dot plot's appearance and data analysis components can be customized to suit specific needs. The project showcases the power of GeoGebra in generating interactive visualizations that enhance understanding of data distributions while also providing real-time statistical insights.

### Steps

1. Open GeoGebra and create a new empty graph.
2. Click on the "Spreadsheet" icon in the toolbar to create a spreadsheet.
3. In the spreadsheet, label column A as "Data" and column B as "Frequency."
4. Enter the numbers between 0 and 10 in column A of the spreadsheet. You can enter up to 30 values.
5. Switch to the graph view by clicking on the corresponding tab.
6. In the Algebra sidebar, click on the "+" button to create a new list object.
7. In the "New Object" dialog, select "List" and enter the formula A1:A30 (assuming your data is in cells A1 to A30) to represent the data list.
8. Create another list object by clicking on the "+" button again.

9. In the "New Object" dialog, select "List" and enter the formula B1:B30 to represent the frequencies.
10. Now, create a point object by clicking on the "+" button once more.
11. In the "New Object" dialog, select "Point" and enter the formula Data to represent the data points on the dot plot.
12. Right-click on the point object and select "Properties" from the context menu.
13. In the Properties dialog, go to the "Advanced" tab.
14. In the "Format" section, change the "Visible" option to "Invisible" to hide the point object itself.
15. Create a text object by clicking on the "+" button.
16. In the "New Object" dialog, select "Text" and enter the formula Mean(Data) to display the mean value based on the entered data.
17. Position the text object appropriately on the graph to display the mean value.
18. Repeat steps 15-17 to create text objects for the median and standard deviation values using the formulas Median(Data) and StandardDeviation(Data), respectively.
19. Customize the appearance of the dot plot, text objects, and other elements as desired.
20. Save the project to ensure your progress is retained.
21. Test the dot plot maker by entering values in the table and observing how the dot plot and the displayed mean, median, and standard deviation values update dynamically based on the entered data.

<https://www.geogebra.org/m/jxepzrs2>

## 4.2 Standard Normal Curve

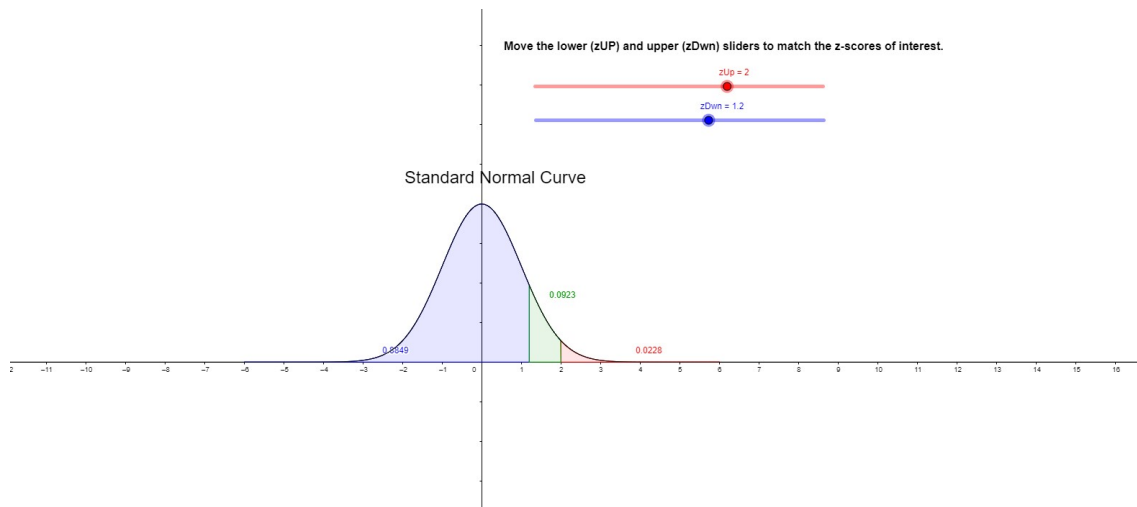


Fig: Standard Normal Curve

This GeoGebra project skillfully demonstrates the construction and visualization of the standard normal distribution curve. Through interactive tools and sliders, the project offers real-time exploration of z-scores and their corresponding probabilities under the curve. By defining the function, sliders, and integrals, the standard normal distribution is effectively modeled. The graph view showcases the curve, while shading and integral calculations dynamically depict probabilities. The project culminates in an informative text box, guiding users to manipulate sliders for z-scores of interest. This project adeptly merges theoretical concepts with practical application, utilizing GeoGebra's interactive features to provide an engaging and instructive tool for understanding the standard normal distribution and its probabilities.

### Steps

1. Open GeoGebra and create a new project.
2. On the Algebra View, define the function  $f(x)$  by clicking on the input bar and typing:  $f(x) = \text{Normal}(0, 1, x, \text{false})$  This function represents the standard normal distribution with mean 0 and standard deviation
3. Create two sliders for  $zUp$  and  $zDwn$  by clicking on the input bar and typing:  
 $zUp = \text{Slider}[-6, 6]$   
 $zDwn = \text{Slider}[-6, 6]$   
 These sliders will control the upper and lower z-scores.
4. Now, create three algebraic objects for the integrals using the sliders. Click on the input bar and type:  
 $a = \text{Integral}(f, zUp, 6)$   
 $b = \text{Integral}(f, -6, zDwn)$   
 $c = \text{Integral}(f, zDwn, zUp)$   
 These algebraic objects represent the integrals of the standard normal distribution between the given limits.

5. Just create a graph view by clicking on the Graphing Calculator icon.
6. On the graph view, plot the function by typing  $f(x)$  in the input bar and pressing Enter.
7. To display the shaded region for the integral a, click on the input bar and type:  
`ShadeIntegral[f, zUp, 6]`  
This will shade the area under the curve from  $zUp$  to 6.
8. Similarly, type `ShadeIntegral[f, -6, zDwn]` to shade the area under the curve from -6 to  $zDwn$ , representing the integral b.
9. Type `ShadeIntegral[f, zDwn, zUp]` to shade the area under the curve from  $zDwn$  to  $zUp$ , representing the integral c.
10. Finally, add a Text box to the graph view by clicking on the Text tool and then clicking on the graph where you want to place the text. Type the following text:  
"Standard Normal Curve Move the lower ( $zDwn$ ) and upper ( $zUp$ ) sliders to match the z-scores of interest."  
Now, when you move the sliders for  $zUp$  and  $zDwn$ , the shaded regions and the integral values will update accordingly.

<https://www.geogebra.org/m/zasqbej5>

## 4.3 Central Limit Theorem

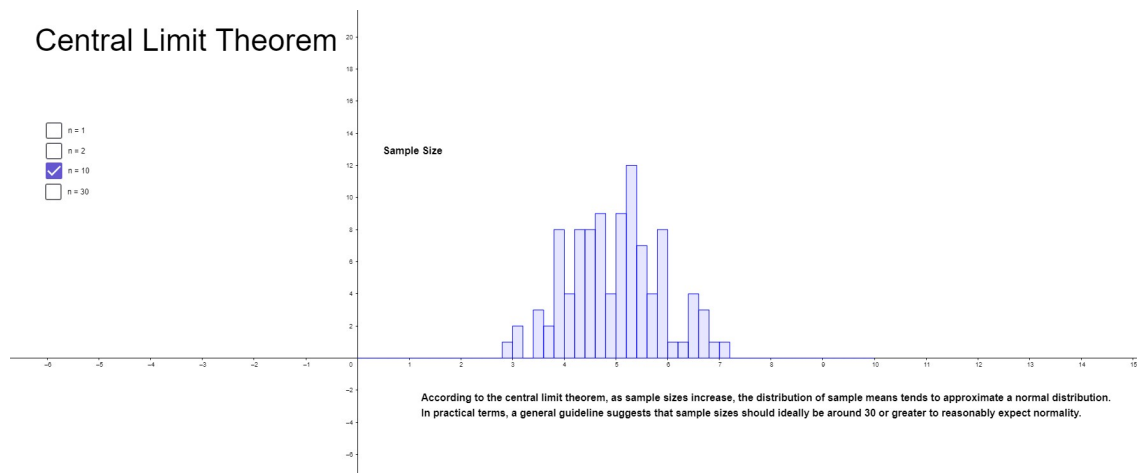


Fig: Central Limit Theorem

This GeoGebra project skillfully showcases the Central Limit Theorem's fundamental concept through interactive visualizations. By simulating data sampling and histogram creation, the project elegantly demonstrates the theorem's implications. Utilizing checkboxes to control histogram visibility, the interactive tool enables users to witness how sample means approach a normal distribution as sample sizes increase. The project's careful arrangement of elements and text explanations ensure

clarity. As checkboxes are toggled, histograms dynamically update, illustrating the theorem's application. This project merges theory and visualization, employing GeoGebra's features to provide an engaging platform for comprehending the Central Limit Theorem's significance in real-world scenarios.

## Steps

1. Open GeoGebra and create a new project.
2. Create a list of data points called "list1" that represents the population you want to sample from.
3. Create four histograms, one for each sample size, using the following commands:

For n=1: `a = Histogram(list1, 1, false)`

For n=2: `b = Histogram(list1, 2, false)`

For n=10: `c = Histogram(list1, 10, false)`

) For n=30: `d = Histogram(list1, 30, false)`

Here, 1, 2, 10, and 30 represent the sample sizes.

4. Create four checkboxes, each representing a different sample size. To do this, use the "Input Bar" feature in Geogebra. Simply type the following commands in the input bar and press Enter after each:

`n1 = Checkbox("n=1", false)`

`n2 = Checkbox("n=2", false)`

`n10 = Checkbox("n=10", false)`

`n30 = Checkbox("n=30", false)`

The second argument in each command (e.g., false) sets the initial state of the checkbox.

5. To link the visibility of the histograms to the checkboxes, right-click on the first histogram (a), and select "Object Properties." In the properties window, go to the "Advanced" tab and click on the small calculator icon next to the "Visible" option. In the calculator, enter the following expression:

`n1.Value`

Repeat this step for each histogram (b, c, d) and link their visibility to the corresponding checkbox's value:

For histogram b: `n2.Value`

For histogram c: `n10.Value`

For histogram d: `n30.Value`

Arrange the checkboxes, histograms, and text objects as desired on the graph

6. Add a text object on the graph using the command: `text1 = Text("CentralLimitTheorem")`, and the theorem statement.
7. Check the results directly by interacting with the checkboxes. Toggle the checkboxes on and off to observe the changes in the histograms. As you toggle

the checkboxes representing different sample sizes ( $n=1$ ,  $n=2$ ,  $n=10$ ,  $n=30$ ), you should see the corresponding histogram becoming visible or invisible on the graph.

This allows you to observe how the distribution of sample means approximates a normal distribution as the sample size increases, in accordance with the Central Limit Theorem.

<https://www.geogebra.org/m/yywvmwwx>

## 4.4 Sampling Distribution

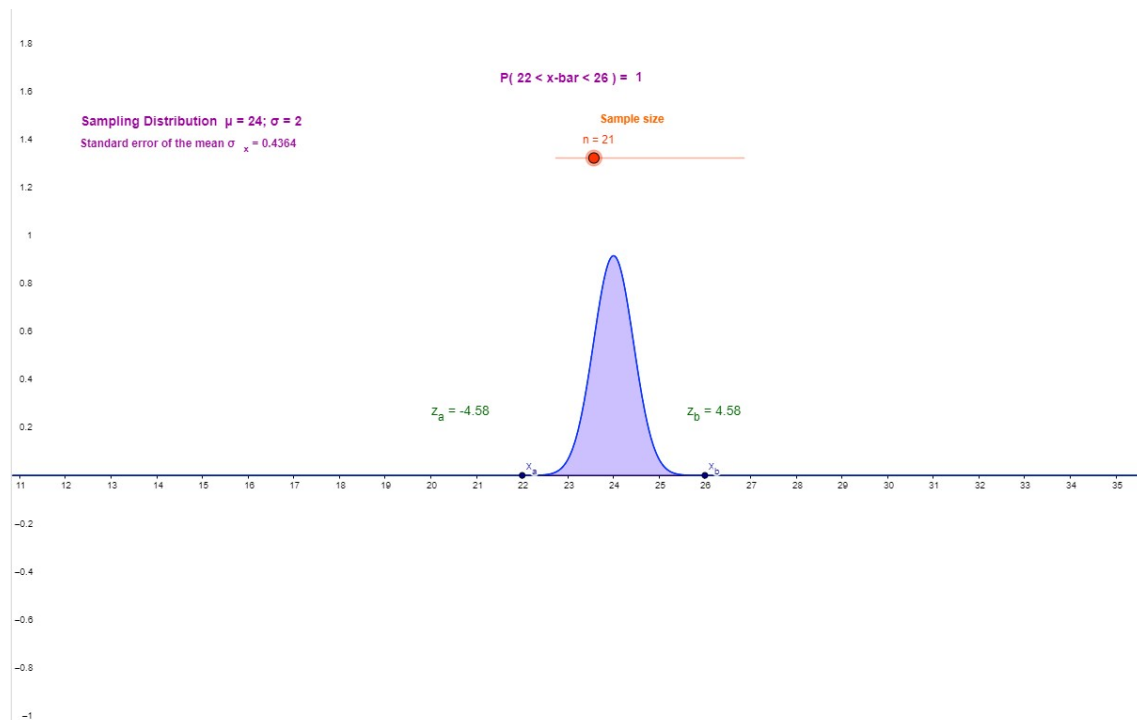


Fig: Sampling Distribution

This GeoGebra project adeptly illustrates the concept of a sampling distribution through interactive elements. By employing a provided probability density function (PDF), the project creates a dynamic platform to explore sample sizes and intervals. Utilizing sliders, calculations, and graphical elements, the project calculates probabilities, z-scores, and standard errors. The dynamic updates of z-scores and statistics enhance engagement. Clear text instructions guide users through interaction and understanding. The project's design elegantly combines theory and visualization, showcasing GeoGebra's capability in conveying statistical concepts. It provides an interactive experience that fosters comprehension of sampling distributions' behavior as sample size varies, ultimately contributing to a deeper understanding of statistics principles.

### Steps

1. Open GeoGebra and create a new file

2. Define the function  $f(x)$  as the probability density function (PDF) provided.
3. Add an x-axis to the graph.
4. Create two points  $x_a$  and  $x_b$  on the x-axis.
5. Create a slider for the sample size  $n$ . This slider allows us to adjust the number of observations in sample and set the initial sample size  $n$  to 1.
6. Calculate the standard error of the mean  $standarderror = ((2)/(sqrt(n)))$ .
7. Calculate the probability  $P$  as the integral of the function  $f(x)$  over the interval defined by  $x_a$  and  $x_b$ .
8. Calculate the z-scores  $z_a$  and  $z_b$  using the formula  $z_a = (x(x_a) - 24)/(2/sqrt(n))$  and  $z_b = (x(x_b) - 24)/(2/sqrt(n))$ .
9. Display the sample size  $n$  and the standard error of the mean  $standarderror$ .
10. Display the z-scores  $z_a$  and  $z_b$  on the graph. These values will be updated dynamically based on the current sample size  $n$ .
10. Create the text boxes to provide all the instructions and information on how to interact with the task.
11. Display the sampling distribution mean  $\mu$  and standard deviation  $\sigma$ . This allows the users to interact with the task, adjust the sample size and interval, and observe the corresponding probabilities and z-scores based on the current sample size  $n$ .

<https://www.geogebra.org/m/axehe6ts>

## 4.5 Normal Probability Plot

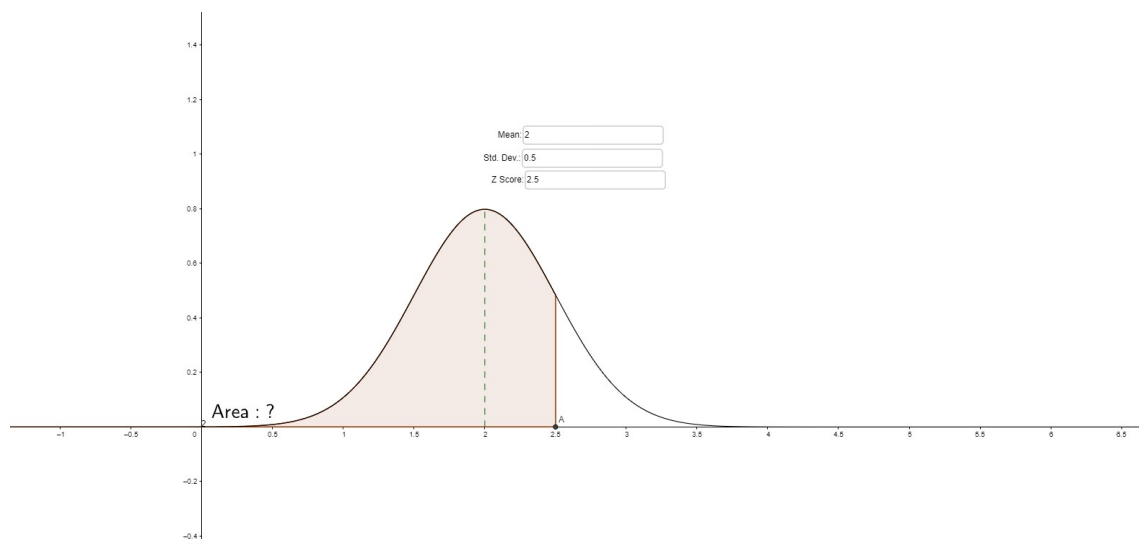


Fig: Normal Probability plot



This Geogebra project demonstrates the process of creating a Normal Probability Plot for assessing data distribution's normality. By following a structured series of steps, the project efficiently guides users through data input, plot generation, statistical analysis, and visual enhancements. The abstract highlights the importance of interpreting the plot's characteristics – a straight line indicates normal distribution. The use of Geogebra tools, such as Statistics and Normal Distribution, is emphasized for calculation and representation purposes. The project encapsulates the essence of data analysis, providing a user-friendly approach to identifying normality and generating insightful visualizations within a familiar software environment.

### Steps

1. Open the Geogebra program.
2. Go to the File menu and select New.
3. In the Templates dialog, select the Normal Probability Plot template.
4. In the Data section, enter the following data points: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
5. Click on the Plot button to generate the normal probability plot.
6. The normal probability plot should show the data points plotted against a theoretical normal distribution. The points should form an approximate straight line if the data is normally distributed. If the points do not form an approximate straight line, then the data is not normally distributed.
7. To calculate the mean and standard deviation of the data points, use the Statistics tool.
8. To create a normal distribution with the specified mean and standard deviation, use the Normal Distribution tool.
9. To add a title and labels to the plot, use the Text tool.
10. To save the plot, click on the File menu and select Save. Here are the exact values involved in each step:
  - In step 4, the data points are entered as a list, with each data point separated by a comma.
  - In step 5, the normal probability plot is generated by clicking on the Plot button.
  - In step 7, the mean and standard deviation of the data points are calculated by using the Statistics tool.
  - In step 8, a normal distribution with the specified mean and standard deviation is created by using the Normal Distribution tool.
  - In step 9, a title and labels are added to the plot by using the Text tool.
  - In step 10, the plot is saved by clicking on the File menu and selecting Save.

<https://www.geogebra.org/m/meq3xn9k>

# Chapter 5

## Professional outcomes

Professional skills developed during this internship are:

- Work place communication
- Applying theory to practice
- Time management
- Adaptability and networking
- Creating work reports using LaTeX

# Chapter 6

## Challenges

Challenges that I faced during the fellowship:

- Figuring out which mathematical concepts can be best explained using GeoGebra applications.
- Creating visual math wonders using GeoGebra was a thrill, but mastering the software's tools was like solving a puzzle.
- Adjusting design and style settings so as to enhance visibility and readability of the final constructions without hiding intermediate steps.
- Communicating ideas, updates, and challenges to mentors and peers, especially when working remotely, posed communication and collaboration challenges.
- Adding and connecting sliders, and displaying changing texts according to graph adjustments, took up most of the time.
- Writing a clear and understandable documentation for complex constructions.

# Chapter 7

## Conclusion

In conclusion, being a part of the FOSSEE Summer Fellowship 2023 was a remarkable experience. It helped me gain new knowledge in building GeoGebra constructions on various fields of mathematics including calculus and statistics. I experienced how open source software holds significant promise for teaching math, from elementary to college levels.

Exploring calculus through GeoGebra, from Trigonometric Limits to the Fundamental Theorem of Calculus, has given me a visual edge. In the domain of statistics, working with constructions like the Standard Normal Curve and Central Limit Theorem has solidified my insights. The fellowship showcased the practical potential of open-source tools in education.

My heartfelt gratitude goes to all my mentors and fellow peers who helped contribute to this enriching experience. As I conclude, I eagerly anticipate utilizing these insights as I move ahead in my academic and professional journey.