

## FOSSEE Summer Fellowship Report

On
Mathematical Applications Project using GeoGebra

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## Chapter 1

## Overview

Spoken Tutorial and Soul project of FOSSEE (Free/Libre and Open Source Software for Education) are projects at Indian Institute of Technology Bombay. They aim to promote the use of Open Source tools throughout India and reduce the use of proprietary software.

During my two month fellowship, I was asked to do Projects for Mathematics using GeoGebra.
I was able to contribute 15 tasks. It was an incredible learning opportunity which I have detailed in this report. I have started with exploring GeoGebra software. I have also mentioned the projects I have worked on. After that, there is a detailed account of my contributions. Finally, I conclude the entire experience.

## Chapter 2

## FOSSEE

### 2.1. FOSSEE Projects

FOSSEE (Free/Libre and Open Source Software for Education) project promotes the use of FLOSS tools to improve the quality of education in our country. It aims to reduce dependency on proprietary software in educational institutions. To ensure commercial software is replaced by equivalent FLOSS tools FOSSEE encourage the use of FLOSS tools through various activities. They also develop new FLOSS tools and upgrade existing tools to meet requirements in academia and research. The FOSSEE project is part of the National Mission on Education through Information and Communication Technology (ICT), Ministry of Education (MoE), Government of India.

### 2.2. SOUL

SOUL (Science OpensoUrce Software for Teaching Learning) is an attempt to put together the popular ICT software used as teaching/learning tools by the community of educators and the learners in basic concept as well as advanced learning of science subjects. These softwares can be used as ICT tools in classroom teaching and learning for topics in science subjects. Learners can learn the software using the resources available for each of these software on the website. They can create a project on a particular topic using the available software. This website can be used as a platform by teachers and students to showcase the projects completed using the software promoted on this page. These projects will be available as resource material for all the interested educators and learners.

The software available for teaching and learning

- ChemCollective Virtual Lab
- Jmol Application
- Avogadro
- Freeplane
- GeoGebra


### 2.3. Mathematical Applications Project using GeoGebra

The mathematics projects using GeoGebra allow students to learn math concepts in a highly visualisable and engaging environment to further deepen their knowledge. There are projects for many concepts including the algebra, theorems, geometry, vectors, graphs etc.

## Chapter 3

## Contributions

### 3.1. Mathematics using GeoGebra

Using the GeoGebra interface, I have created various mathematical projects:

- Finding the definite integral and Riemann Sums
- Matrix transformation of $3 \times 3$ matrix
- Contour
- Spinning parametric curves
- Conic section
- Rotation of conics
- Revolution of conic surfaces
- Butterfly curve


### 3.1.1. Finding the definite integral and Riemann Sums

Finding Definite integral, left Riemann sum, right Riemann sum and middle Riemann rule by the following steps.

1. Type the function $\mathbf{f}(\mathbf{x})=\boldsymbol{\operatorname { s i n }}(\mathbf{x})$ in the input text box.
2. Create three sliders $\mathbf{a}, \mathbf{b}$, and $\mathbf{n}$. We can choose lower limit, upper limit and number of rectangles as per our convenience.
3. Here slider $\mathbf{a}$ is used for lower limit, slider $\mathbf{b}$ for upper limit and slider $\mathbf{n}$ for the number of rectangles.
4. Using input box tool let us create input boxes by clicking inside the Graphics view. In linked objects, let us select the input as $\mathbf{f}(\mathbf{x}), \mathbf{a}, \mathbf{b}, \mathbf{n}$.
5. Type the integral command in input bar as "integral(f ,a , b)" and press Enter to show the definite integral.
6. To find the left Riemann sum use leftsum command. Type the input as "leftsum(f,a,b,n)"
7. In GeoGebra we do not have commands to find right Riemann sum and middle Riemann sum so we use $\Delta \mathbf{x}$. Where $\Delta \mathbf{x}$ is the width of each rectangle.
8. To find $\Delta \mathbf{x}$ by type " $\Delta \mathbf{x}=(\mathbf{a}-\mathbf{b}) / \mathbf{n}$ " in the input bar.
9. To find middle Riemann sum and right Riemann sum, leftsum command along with $\Delta \mathbf{x}$ is used.
10. To find middle Riemann sum type input as " $\operatorname{LeftSum}(\mathbf{f}(\mathbf{x}+\boldsymbol{\Delta x} / \mathbf{2}), \mathbf{a}, \mathbf{b}, \mathbf{n})$ " and to find right Riemann sum as " $\operatorname{LeftSum}(\mathbf{f}(\mathbf{x}+\Delta \mathbf{x}), \mathbf{a}, \mathbf{b}, \mathbf{n})$ " .
11. Drag the sliders to see differences in the graph.
12. You can colour the variables to differentiate them in the graph.


### 3.1.2. Matrix transformation of 3x3 matrix

A $3 \times 3$ matrix can be transformed with respect to basis using GeoGebra by the following steps.

1. The elements $\mathrm{a} 11, \mathrm{a} 21, \mathrm{a} 31, \mathrm{a} 21, \mathrm{a} 22, \mathrm{a} 23, \mathrm{a} 31, \mathrm{a} 32$ and a 33 are created using slider tool and input text box tool.
2. Create $3 \times 3$ matrix by typing " $\mathrm{A}=\left(\begin{array}{lll}a 11 & a 12 & a 13 \\ a 21 & a 22 & a 23 \\ a 31 & a 32 & a 33\end{array}\right)$,"
3. Using slider tool and input box tool create vectors $\mathrm{u}, \mathrm{v}$ and w using convenient values.
4. All the sliders created and input box created can be seen in the Graphics view.
5. Type in input as "V = Vector ( $\mathbf{u}, \mathbf{v}, \mathbf{w})$ )"
6. To find the matrix transformation type as " $\mathbf{T}=((\mathbf{A V}))$ " where A is the matrix and V is the vector.
7. Drag the slider to see the difference in the graph.
8. We can see the matrix transformation as there is a change in the values of vectors.


### 3.1.3. Contour

A contour line or a level curve for a given surface is the curve of intersection of the surface with a horizontal plane, $\mathrm{z}=\mathrm{c}$. A collection of contour lines, projected on the xy-plane, is a contour plot of the surface.

For any desired function contour plot can be drawn using GeoGebra.
Steps

1. Type the function $\mathbf{f}(\mathbf{x})=\boldsymbol{\operatorname { s i n }}(\mathbf{x}) \boldsymbol{\operatorname { c o s }}(\mathbf{y})$ in the input text box.
2. Create two sliders $\mathbf{a}$ and $\mathbf{A}$. Set $\mathbf{a}$ as the increment of $A$ value.
3. By the definition of contour line type in input bar as "eq1:f = A" .
4. We can see the contour pattern in the Graphics view as the sliders moves.
5. In 3 D view we could see the contour formed by the given equation.


### 3.1.4. Spinning parametric curves

A parametric curve is defined by its corresponding parametric equations and within a given interval $t$.
For a given parametric function we can visually see the spinning of parametric curve using GeoGebra.
Steps:

1. Create a slider called "spin" using the slider tool.
2. Type in input bar as "a= Curve $\left(\cos ^{\wedge}(2)(t), \sin \wedge(2)(t), t, t, 0,2 \pi\right)$ "
3. To spin the parametric curves with respect to $X$ axis type in input bar as "b=Surface(a,spin,xAxis)"
4. For Y axis type as "Surface(a,spin,yAxis)" and for Z axis type in input bar as "Surface(a,spin,zAxis)"
5. Drag the spin slider to view the spinning of the parametric curve with respect to $\mathrm{X}, \mathrm{Y}$ and Z axis.

( $\begin{aligned} & \text { spin }=6.65 \\ & 0 \longrightarrow 9.42 \quad \text { © }\end{aligned}$
$a=$ Curve $\left(\cos ^{2}(t), \sin ^{2}(t), t, t, 0,2 \pi\right)$
$x=\cos ^{2}(t)$
$\left.=\begin{array}{l}y=\sin ^{2}(t) \\ z=t\end{array}\right\} 0 \leq t \leq 6.28$
$\mathrm{b}=$ Surface $(\mathrm{a}$, spin,$\times$ Axis $)$
$=\left(\begin{array}{c}\cos ^{2}(u) \\ \sin ^{2}(u) \cos (v)+u(-\sin (v)) \\ \sin ^{2}(u) \sin (v)+u \cos (v)\end{array}\right)$
$\mathrm{c}=$ Surface $(\mathrm{a}$, spin, $\mathrm{y} A \mathrm{xis})$
$=\left(\begin{array}{c}\cos ^{2}(u) \cos (v)+u \sin (v) \\ \sin ^{2}(u) \\ \cos ^{2}(u)(-\sin (v))+u \cos (v)\end{array}\right)$
$\mathrm{d}=$ Surface(a, spin, zAxis )
$=\left(\begin{array}{c}\cos ^{2}(u) \cos (v)+\sin ^{2}(u)(-\sin (v)) \\ \cos ^{2}(u) \sin (v)+\sin ^{2}(u) \cos (v) \\ u\end{array}\right)$

+ Input...



### 3.1.5. Conic sections in 3D

A conic section, or a quadratic curve is a curve obtained from a cone's surface intersecting a plane.
We can draw conic sections using GeoGebra 3D Graphics view by following these steps.

1. Let us create the sliders $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{p}, \mathrm{q}$ and r using the slider tool.
2. Type in the input bar "d: InfiniteCone((a,b,c),(p,q,r),(( $\pi) /(3)))$ " .
3. To create an intersection plane use "Plane through 3 points" tool in the 3D Graphics view.
4. Click any three points say A, B and C in the created infinite cone, the intersection plane will be created.
5. Using A, B and C points of intersection plane we can see the different conic sections by dragging them.
6. To view the conic sections clearly in the 3 D view let us type in the input bar as " f :

IntersectPath(p_\{1\},d)" where p 1 is the intersection plane and d is the infinite cone created.
7. By dragging the points $\mathrm{A}, \mathrm{B}$ and C we can view the four types of conic sections such as circle, ellipse, parabola and hyperbola in the 3D view.


### 3.1.6. Rotation of conics

We can visually experience the rotation of the conic sections using GeoGebra by the following steps.

1. Let us create a circle with radius 5 units using circle: Center \& Radius tool.
2. Click a point in the Graphics view, and type radius as 5 units and click Ok, a circle of center point A and radius 5 units is drawn.
3. To draw an ellipse use the ellipse tool and click three points in the Graphics view according to your convenience.
4. To draw a parabola use the parabola tool, click a point and then click yaxis in the Graphics view.
5. To draw a hyperbola use the hyperbola tool, and click three points on the Graphics view.
6. Use the object properties for each object and colour the graphs to distinguish them clearly.
7. Let us create an angle slider $\alpha$ so that the conics will rotate with respect to $\alpha$.
8. To rotate the created conics, use the Rotate command.
9. Type in the input bar as " c ': Rotate ( $\mathrm{c}, \alpha$ )" where c is the circle created and c ' is the circle rotated by angle $\alpha$.
10. Type in the input bar as "d': Rotate(d, $\alpha$ )" where $d$ is the ellipse created and d" is the ellipse rotated by angle $\alpha$.
11. Type in the input bar as "e': Rotate $(e, \alpha)$ " where e is the parabola created and $e$ ' is the parabola rotated by angle $\alpha$.
12. Type in the input bar as " $f$ ': Rotate $(f, \alpha)$ " where $f$ is the hyperbola created and $f$ ' is the hyperbola rotated by angle $\alpha$.
13. To trace the rotating conics, select the trace on option from the settings of each conic rotated by angle $\alpha$ given in the input bar.
14. Create check boxes for each conics using check box tool in the Graphics view.
15. Select animation option in the slider of angle $\alpha$ and select the conic from the check box thus we can view the rotation of the conics in both the Graphics and 3D view.


### 3.1.7. Revolution of conic surfaces

Surfaces formed by the conic sections can be created and revolution of the conic sections could be experienced visually in GeoGebra by the following steps.

1. Draw a circle with radius 5 units using circle: Center \& Radius tool.
2. Click a point in the Graphics view the and type radius as 5 and Click OK, circle of center point A and radius 5 units is drawn.
3. To draw an ellipse use the ellipse tool and click three points in the Graphics view according to convenience.
4. To draw a parabola use the parabola tool, click a point and then click yaxis in the Graphics view.
5. To draw a hyperbola use the hyperbola tool. and click three points on the Graphics view.
6. Use the object properties for each object and colour the graphs to distinguish them clearly.
7. Let us create an angle slider $\alpha$, so that the conics will rotate with respect to $\alpha$.
8. For the revolution of the conics about an axis use Surface of Revolution tool in the 3D view and then click the curve of the conic to bring about the revolution.
9. Input will be automatically generated after clicking the tool and the object. We can also type the inputs in the input bar and press Enter as follows.
10. Type in the input bar as " $c$ '=Surface( $c, \alpha, x$ Axis)" where $c$ is the circle created and $c$ ' is the revolution of circle with angle $\alpha$.
11. Type in the input bar as " $d$ '=Surface( $\mathrm{d}, \alpha, \mathrm{xAxis)}$ " where d is the ellipse created and d ' is the revolution of ellipse with angle $\alpha$.
12. Type in the input bar as "e'=Surface(e, $\alpha, x A x i s)$ " where e is the parabola created and e ' is the revolution of parabola with angle $\alpha$.
13. Type in the input bar as " $f$ ' $=$ Surface( $f, \alpha, x$ Axis)" where $f$ is the hyperbola created and $f$ ' is the revolution of hyperbola with angle $\alpha$.
14. To trace the revolution of the conics, select the trace on option from the settings of each input of revolution of the conics given in the input bar.
15. Create check boxes for each conics using check box tool in the Graphics view.
16. Select animation option in the slider for angle $\alpha$ and select the conic from the check box thus we can view the revolution of the conics in both the Graphics and 3D view.


### 3.1.8. Butterfly curve

One of the most fascinating mathematical equations is the use of polar equations that produces a graph which looks like a butterfly.

Let see the visual treat using GeoGebra.

1. Create slider $t$ which will be used in the butterfly equation as a variable.
2. The equation for the fascinating butterfly curve is as shown. Type "B=(sin(t) $\left(e^{\wedge}(\cos (t))-2 \cos (4 t)\right.$ $\left.+\sin \wedge(5)(((t) /(12)))), \cos (t)\left(e^{\wedge}(\cos (t))-2 \cos (4 t)+\sin \wedge(5)(((t) /(12)))\right)\right) "$.
3. To change the value of $t$, type "a=Curve(sin(t) ( $\left.\left.e^{\wedge}(\cos (t))-2 \cos (4 t)+\sin \wedge(5)((t) /(12))\right)\right), \cos (t)$ $\left.\left(e^{\wedge}(\cos (\mathrm{t}))-2 \cos (4 \mathrm{t})+\sin ^{\wedge}(5)(((\mathrm{t}) /(12)))\right), \mathrm{t}, 0,10\right) "$ ( set up $\left.0 \leq \mathrm{t} \leq 10\right)$.
4. Type "b=Curve(sin $\left.(\mathrm{t})\left(e^{\wedge}(\cos (\mathrm{t}))-2 \cos (4 \mathrm{t})+\sin ^{\wedge}(5)((\mathrm{t}) /(12))\right)\right), \cos (\mathrm{t})\left(e^{\wedge}(\cos (\mathrm{t}))-2 \cos (4 \mathrm{t})\right.$ $+\sin \wedge(5)(((t) /(12)))), t, 10,20) "($ setting up $10 \leq t \leq 20)$ press Enter.
5. Type "c=Curve (sin $(\mathrm{t})\left(\boldsymbol{e}^{\wedge}(\cos (\mathrm{t}))-2 \cos (4 \mathrm{t})+\sin \wedge(5)(((\mathrm{t}) /(12)))\right), \cos (\mathrm{t})\left(e^{\wedge}(\cos (\mathrm{t}))-2 \cos (4 \mathrm{t})\right.$ $+\sin \wedge(5)(((t) /(12)))), t, 20,30) "$ (setting up $20 \leq t \leq 30)$ press Enter.
6. Type "d=Curve $\left(\sin (\mathrm{t})\left(\boldsymbol{e}^{\wedge}(\cos (\mathrm{t}))-2 \cos (4 \mathrm{t})+\sin \wedge(5)(((\mathrm{t}) /(12)))\right), \cos (\mathrm{t})\left(e^{\wedge}(\cos (\mathrm{t}))-2 \cos (4 \mathrm{t})\right.\right.$ $+\sin \wedge(5)(((t) /(12)))), t, 30,40) "$ (setting up $30 \leq t \leq 40)$ press Enter.
7. Use the Setting menu for each object and colour the curve to distinguish them clearly.
8. From the butterfly curve equations, we formed in the 3D Graphics we could see the animation of the moving point B forming butterfly curve.


### 3.2. Science models using GeoGebra

- RC circuit
- RL circuit
- RLC Resonance
- Electromagnetic field
- Electric potential
- DNA model
- DNA replication model


### 3.2.1. RC Circuit

A resistor-capacitor circuit (RC Circuit) is an electrical circuit consisting of passive components like resistors and capacitors, driven by the current source or the voltage source.

Let us plot the RC circuit diagram using GeoGebra software.

1. Create two sliders for $r$ for resistance and $c$ for capacitance.
2. Type " $\mathrm{R}=10^{\mathrm{r}}$ and $\mathrm{C}=10^{\mathrm{c}}$ " so that the values of resistance and capitance are in terms of powers of 10 .
3. Give values for initial current flow at time $\mathrm{t}=0$ as v 0 and current flow at time $\mathrm{t}=1$ as v 1 by creating sliders. The current flow through DC battery as Vs .
4. Here we will use the if commands to change the current flow in the circuit.
5. Type "V_\{s\}(t)=If(t¥0, v_\{1\}, v_\{0\})" using if command.
6. The current flow in the resistor and capacitor is denoted as VR and VC .
7. Type as "V_\{R\}(t)=If(t¥0,(v_\{1\}-v_\{0\}) $\left.e^{\wedge}(((-t) /(\tau))), 0\right)$ " and press Enter.
8. Type as " V_\{C\}(t)=If(t $\geq 0$, $v_{-}\{1\}+\left(v_{-}\{0\}-v_{-}\{1\}\right) \boldsymbol{e}^{\wedge}(((-\mathrm{t}) /(\mathrm{T})))$, $\left.\mathrm{v}_{-}\{0\}\right)$ " and press Enter.
9. Now we get the RC circuit curve for given R and C values.
10. We know that $\tau$ is the time constant at $t=1$ we get $\tau=1$ type as $\tau: x=1$.
11. Maximum charge of capacitor is attained at 5 -time constants ( $5 \tau$ ).
12. By changing the resistance and capacitance values we could see the change of current flow in RC circuit.


### 3.2.2. RL circuit

One of the fundamental circuits RL circuits can be observed with change in current. We can demonstrate them using GeoGebra software.

1. Create two sliders for $r$ for resistance and $l$ for inductance.
2. Type " $\mathrm{R}=10^{\mathrm{r}}$ and $\mathrm{L}=10^{1}$ " so that the resistance and inductance values are given.
3. Give values for initial current flow at time $t=0$ as $v 0$ and current flow at time $t=1$ as $v 1$ by creating sliders. The current flow through DC battery as Vs .
4. Here we will use the if commands to change the current flow in the circuit.
5. Type "V_\{s\}(t)=If(t $\left.\mathrm{t} 00, \mathrm{v}_{-}\{1\}, \mathrm{v}_{-}\{0\}\right)$ "
6. The current flow in the resistance and inductor is denoted as VR and VL .
7. Type as "V_\{L\}(t)=If(t $\geq 0$, $\left.\left(v_{-}\{1\}-v_{-}\{0\}\right) e^{\wedge}(((-t) /(\tau))), 0\right)$ " and press Enter.
8. Type as " $\mathrm{V}_{-}\{\mathrm{R}\}(\mathrm{t})=\mathrm{If}\left(\mathrm{t} \geq 0\right.$, $\mathrm{v}_{-}\{1\}+\left(\mathrm{v}_{-}\{0\}-\mathrm{v}_{-}\{1\}\right) \boldsymbol{e} \wedge(((-\mathrm{t}) /(\mathrm{T})))$, $\left.\mathrm{v}_{-}\{0\}\right)$ " and press Enter.
9. Now we get the RL circuit curve for given $R$ and $L$ values.
10. We know that $\tau$ is the time constant at $t=1$ we get $\tau=1$ type as $\tau: x=1$.
11. Maximum charge of capacitor is attained at 5 -time constants ( $5 \tau$ ).
12. By changing the resistance and inductance values we could see the change of current flow in RL circuit.


### 3.2.3. RLC Circuit

A fundamental circuit RLC circuit can be demonstrated to show the change in resistance, capacitance and inductance. Let us see the RLC resonance by the following steps.

1. Create two sliders, one for maximum voltage across the RLC circuit as V and other for frequency as $f$ by giving desired values.
2. In $A C$ circuit it produces an emf of $u(t)=V \sin (t)$ we know that $\omega=2 \pi f$.
3. Type $\omega=2 \pi f$ and then type $u(t)=V \sin (t)$ we get out emf graph.
4. Create sliders for resistance, inductance and capacitance as $\mathrm{R}, \mathrm{L}$ and C with desired values.
5. Since the elements are in series, the same current flows through each element at all times. The relative phase between the current and the emf is not obvious when all three elements are present. We have the formula for current as $\mathrm{i}(\mathrm{t})=\mathrm{I} \sin (\omega \mathrm{t}-\phi)$ where I is the maximum current or current amplitude.
6. Here $\varphi$ is phase angle between the current and the applied voltage we know $\phi=\tan \wedge-1((\mathrm{XC}-\mathrm{XL}) / \mathrm{R})$.
7. Type as "X_\{C\}=((1)/(2 $\left.\left.\pi \mathrm{f} \mathrm{C}^{*} 10 \wedge(-6)\right)\right)$ " and "X_\{L\}=2$\pi \mathrm{f} \mathrm{L}$ " press Enter, to calculate XC and XL. Since C values are taken in $\mu$ we get $10^{-6} \mathrm{in}$ the formula.
8. To find I we have $\mathrm{I}=\mathrm{V} / \mathrm{Z}$ where Z is the impedance of the circuit.
9. Type "Z=sqrt(R^(2)+X_\{C\}-X_\{L\})" and "I=((V)/(Z))*10^(3)" press Enter.

Since I is in Ampere, we convert watts to Ampere by multiplying 103 in the formula.
10. Type " $\phi=\tan \wedge(-1)\left(\left(\left(X \_\{C\}-X \_\{L\}\right) /(R)\right)\right)$ " and "i(t)=I $\sin (\omega t-\phi)$ " press Enter. We get the current flow graph.
11. The resultant graph is the resonance graph of RLC circuit.


### 3.2.4. Electromagnetic wave

We shall graphically demonstrate the Electromagnetic waves created as the result of vibrations between an electric and a magnetic field using GeoGebra software.

1. Create sliders for $A$ and $B$ where $A$ represents arrow present in one oscillation and $B=2 A$.
2. Give value for wavelength $\lambda$ by creating slider where $\lambda$ is the distance covered by one complete cycle of the wave.
3. Type as " $\mathrm{dx}=((\lambda) /(\mathrm{A}))$ " where dx is the change due to magnetic flux.
4. In input bar type " $\mathrm{k}=2 *((\pi) /(\lambda))$ " where k is the number of wavelengths that fit into a distance of $2 \pi$ in the units being used.
5. Let $P$ be the sequence of points to represent arrows in the propagating waves. Type " $\mathrm{P}=$ Sequence ( $(\mathrm{k} \mathrm{dx}, 0), \mathrm{k}, 0, \mathrm{~B})$ " and press Enter.
6. The period of oscillation $t$ is the time required for a complete oscillation. The angular frequency $\omega$. The electric field can be represented as ef.
7. Create sliders for $\omega$, ef and t .
8. Type " $f(x, t)=\sin (\omega t-k x)$ " from the formula of electromagnetic wave.
9. To see the arrow representation of the electric and magnetic field we create sequences using electric flux.
10. Type as "Ef=Sequence((n dx,0,ef f(n dx,t)),n,0,B)" and "Eff=Sequence(Vector(P(i),Ef(i)),i,0,B)" to represent electric waves.
11. Type as "Bf=Sequence(( $n$ dx,-ef $f(n d x, t), 0), n, 0, B)$ " and "Bff=Sequence(Vector(P(i),Bf(i)),i,0,B)" to represent magnetic waves.
12. Thus, the resultant 3D graphics show the electromagnetic wave propagation.


### 3.2.5. Electric potential

Due to the movement of two charges we can experience the electric potential in 3D Graphics using GeoGebra.

1. Let q1 and q2 be the point charges at a distance of X 1 and X 2 .
2. Consider Q1 and Q2 be the quantity of charges q 1 and q 2 .
3. Create sliders for X1, X2, Q1 and Q2 by giving desired values.
4. Type q 1 and q 1 position of charges in input bar as " $\mathrm{q} 1=(\mathrm{X} 1,0)$ " and " $\mathrm{q} 2=(\mathrm{X} 2,0)$ ".
5. To find the electric potential energy of a point charge we have formula for total electric potential we can add them all.
6. Type as "E1(x,y)=((Q1)/((y-y(q1))^(2)+(x-x(q1))^(2)))" in input bar to get the electric potential of q1.
7. To find electric potential of q 2 type as "E2(x,y)=((Q2)/((y-y(q2))^(2)+(x-x(q2))^(2)))".
8. The total electric potential energy is found by adding both E1 and E2. Thus, type as " $\mathrm{E}(\mathrm{x}, \mathrm{y})=\mathrm{E} 1(\mathrm{x}, \mathrm{y})+\mathrm{E} 2(\mathrm{x}, \mathrm{y})$ " press Enter.
9. In 3D Graphics we can see the changes in position of the charges affecting the electric potential energy.


### 3.2.6. DNA Model

Deoxyribonucleic Acid is a polymer composed of two polynucleotide chains that coil around each other to form a double helix.

1. Create sliders for twists in DNA, backbone of DNA, bases of DNA and spin as $\mathrm{T}, \mathrm{Bb} 1, \mathrm{Bb} 2, \mathrm{~b} 1$, b 2 and S . Where Bb 1 and Bb 2 represent the strands of DNA. Bases of those strands are given by b1 and b2.
2. To give base pairs in the strands we create points sequence by giving input as
"Bb1p=Sequence(Sphere(( $\cos (\mathrm{T} i \pi+S), \sin (\mathrm{T} i \pi+S), \mathrm{i}), 0.25), \mathrm{i}, 0, \mathrm{~b} 1)$ " and
"Bb2p=Sequence(Sphere(( $\cos (\mathrm{T}$ i $\pi+\pi+\mathrm{S}), \sin (\mathrm{T} \mathbf{i} \pi+\pi+\mathrm{S}), \mathrm{i}), 0.25), \mathrm{i}, 0, \mathrm{~b} 1)$ ".
3. We use curve tool in GeoGebra to create strands. Type "BB1=Curve $(\cos (\mathrm{t} T \pi+\mathrm{S}) \sin (\mathrm{t} T$ $\pi+S), \mathrm{t}, \mathrm{t}, 0, \mathrm{Bb} 1)$ " and "BB2=Curve( $\cos (\mathrm{t} T \pi+\pi+\mathrm{S}), \sin (\mathrm{t} T \pi+\pi+\mathrm{S}), \mathrm{t}, \mathrm{t}, 0, \mathrm{Bb} 2)$ " press enter to see the strands ford in 3D graphics.
4. To differentiate the nitrogen bases we can create points sequence in the middle of strand by
 $\pi+\pi+S), i)$ ), $, 0, \mathrm{~b} 1)$ " in input bar.
5. Let us show 4 bases in a strand by inputs of n1,n2, n3 and n4 in terms of b1. In another strand we could create bases by giving inputs $\mathrm{n} 5, \mathrm{n} 6, \mathrm{n} 7$ and n 8 in terms of b 2 .
6. Type in input bar as " $\mathrm{n} 1=$ Sequence(Segment $\left(\left(\cos \left(\mathrm{T}^{*} 4 \mathrm{i} \pi+\mathrm{S}\right), \sin (\mathrm{T} * 4 \mathrm{i} \pi+\mathrm{S}), 4\right.\right.$ i),Midpoint(( $\left.\cos \left(T^{*} 4 \mathrm{i} \pi+S\right), \sin \left(\mathrm{T}^{*} 4 \mathrm{i} \pi+S\right), 4 \mathrm{i}\right),\left(\cos \left(\mathrm{T}^{*} 4 \mathrm{i} \pi+\pi+\mathrm{S}\right), \sin \left(\mathrm{T}^{*} 4 \mathrm{i} \pi+\pi+\mathrm{S}\right), 4\right.$ i)),i,,0,b1)".
7. Type in input bar as " $n 5=$ Sequence(Segment ( $\cos (T * 4 \mathrm{i} \pi+\pi+\mathrm{S}), \sin (T * 4 \mathrm{i} \pi+\pi+\mathrm{S}), 4$ i),Midpoint(( $\left.\cos \left(\mathrm{T}^{*} 4 \mathrm{i} \pi+\pi+\mathrm{S}\right), \sin \left(\mathrm{T}^{*} 4 \mathrm{i} \pi+\pi+\mathrm{S}\right), 4 \mathrm{i}\right),\left(\cos (\mathrm{T} * 4 \mathrm{i} \pi+\mathrm{S}), \sin \left(\mathrm{T}^{*} 4 \mathrm{i} \pi+\mathrm{S}\right), 4\right.$ i)),i, $, 0, b 2$ )".
8. Taking next terms of 4 i in input we could create inputs for $\mathrm{n} 2, \mathrm{n} 3, \mathrm{n} 4, \mathrm{n} 6, \mathrm{n} 7$ and n 8 .
9. Give suitable colour to the bases and move slider or animation of $S$ that is spin.
10. In 3D Graphics we could see the DNA model spinning.


### 3.2.7. DNA replication model

Two DNA strands replicating two new DNA can be demonstrated using GeoGebra software.

1. Create sliders for twists in DNA, backbones of DNA, bases of DNA, spin and replication of DNA as T, Bb1, Bb2, b1,b2, S and R. Where Bb1 and Bb2 represent the strands of DNA. Bases of those strands are given by b1 and b2.
2. To give base pairs in the strands we create points sequence by giving input as "Bb1p=Sequence(Sphere(( $\cos (\mathrm{T} i \pi+S), \sin (\mathrm{T} i \pi+S), \mathrm{i}), 0.25), \mathrm{i}, 0, \mathrm{~b} 1)$ " and "Bb2p=Sequence(Sphere(( $\cos (\mathrm{T}$ i $\pi+\pi+\mathrm{S}), \sin (\mathrm{T}$ i $\pi+\pi+\mathrm{S}), \mathrm{i}), 0.25), \mathrm{i}, 0, \mathrm{~b} 1)$ ".
3. To differentiate the nitrogen bases we can create points sequence in the middle of strand by typing "Np=Sequence(Midpoint(( $\cos (\mathrm{T} i \pi+S), \sin (\mathrm{Ti} \pi+S), \mathrm{i}),(\cos (\mathrm{T} i \pi+\pi+S), \sin (\mathrm{T} \mathrm{i}$ $\pi+\pi+\mathrm{S}), \mathrm{i}), \mathrm{i}, 0, \mathrm{~b} 1)$ " in input bar.
4. Let us say P and Qare the strands of the DNA to be replicated. Type in input as " $\mathrm{P}=\mathrm{Curve}(\cos (\mathrm{t} T$ $\pi+\pi+S), \sin (t \mathrm{~T} \pi+\pi+S), t, t, 0, B b 1)$ " and "Q=Curve $(\cos (\mathrm{t} T \pi+S), \sin (\mathrm{t} T \pi+\mathrm{S}), \mathrm{t}, \mathrm{t}, 0, \mathrm{Bb} 1)$ "
5. To show the replicated DNA strands we can give inputs as "P1=Curve ( $2 \mathrm{R}+\cos (\pi \mathrm{t}+\pi+\mathrm{S}), \sin (\pi \mathrm{t}$ $\mathrm{T}+\pi+\mathrm{S}), \mathrm{t}, \mathrm{t}, 0, \mathrm{Bb} 1)$ " and " $\mathrm{P} 2=\mathrm{Curve}(2 \mathrm{R}+\cos (\mathrm{t} T \pi+\mathrm{S}), \sin (\mathrm{t} T \pi+\mathrm{S}), \mathrm{t}, \mathrm{t}, 0, \mathrm{Bb} 2)$ " press Enter.
6. For Q also we use the same type of inputs such as "Q1=Curve $(-2 \mathrm{R}+\cos (\pi \mathrm{t} \mathrm{T}+\mathrm{S}), \sin (\pi \mathrm{t}$ $\mathrm{T}+\mathrm{S}), \mathrm{t}, \mathrm{t}, 0, \mathrm{Bb} 1)$ " and "Q2=Curve(-2R+cos( $\pi \mathrm{t} \mathrm{T}+\pi+\mathrm{S}), \sin (\pi \mathrm{t} \mathrm{T}+\pi+\mathrm{S}), \mathrm{t}, \mathrm{t}, 0, \mathrm{Bb} 2)$ ".
7. The replicated base pairs can be shown by creating inputs as " $\mathrm{Pb} 11=$ Sequence(Segment((-2 $\mathrm{R}+\cos (\mathrm{T} * 4 \mathrm{i} \pi+\mathrm{S}), \sin (\mathrm{T} * 4 \mathrm{i} \pi+\mathrm{S}), 4 \mathrm{i})$, Midpoint( $(-2 \mathrm{R}+\cos (\mathrm{T} * 4 \mathrm{i} \pi+\mathrm{S}), \sin (\mathrm{T} * 4 \mathrm{i} \pi+\mathrm{S}), 4 \mathrm{i}),(-2$ $\mathrm{R}+\cos (\mathrm{T} * 4 \mathrm{i} \pi+\pi+\mathrm{S}), \sin (\mathrm{T} * 4 \mathrm{i} \pi+\pi+\mathrm{S}), 4 \mathrm{i}))$ ), $\mathrm{i}, 0, \mathrm{~b} 1)$ " and "Pb12=Sequence(Segment((-2 $R+\cos (T * 4 \mathrm{i} \pi+\pi+S), \sin (T * 4 \mathrm{i} \pi+\pi+\mathrm{S}), 4 \mathrm{i})$, Midpoint( $(-2 \mathrm{R}+\cos (\mathrm{T} * 4 \mathrm{i} \pi+\pi+\mathrm{S}), \sin (\mathrm{T} * 4 \mathrm{i} \pi+\pi+\mathrm{S}), 4$ i),(-2 R+cos(T*4i $\left.\left.\pi+S), \sin \left(T^{*} 4 i \pi+S\right), 4 i\right)\right)$ ), $\left., 0, b 2\right)$ ".
8. The replicated base pairs can be shown by creating inputs as "Pb11=Sequence(Segment((-2 $\mathrm{R}+\cos (\mathrm{T} * 4 \mathrm{i} \pi+\mathrm{S}), \sin (\mathrm{T} * 4 \mathrm{i} \pi+\mathrm{S}), 4 \mathrm{i}), \mathrm{Midpoint}\left(\left(-2 \mathrm{R}+\cos \left(\mathrm{T}^{*} 4 \mathrm{i} \pi+\mathrm{S}\right), \sin (\mathrm{T} * 4 \mathrm{i} \pi+\mathrm{S}), 4 \mathrm{i}\right),(-2\right.$ $\mathrm{R}+\cos (\mathrm{T} * 4 \mathrm{i} \pi+\pi+\mathrm{S}), \sin (\mathrm{T} * 4 \mathrm{i} \pi+\pi+\mathrm{S}), 4 \mathrm{i}))$ ), $\mathrm{i}, 0, \mathrm{~b} 1)$ " and "Pb12=Sequence(Segment((-2 $R+\cos (T * 4 i \pi+\pi+S), \sin (T * 4 i \pi+\pi+S), 4 i), M i d p o i n t((-2 R+\cos (T * 4 i \pi+\pi+S), \sin (T * 4 i \pi+\pi+S), 4$ i),(-2 R+cos(T*4i $\pi+S), \sin (T * 4 i \pi+S), 4 i)), i, 0, b 2) "$.
9. Where Pb 11 and Pb 12 are the base pairs of P strand likewise we can add next terms of $4 i$ to get remaining base pairs of P such as $\mathrm{Pb} 21, \mathrm{~Pb} 22, \mathrm{~Pb} 31, \mathrm{~Pb} 32, \mathrm{~Pb} 41$ and Pb 42 .
10. Where Qb11 and Qb12 are the base pairs of Q strand likewise we can add next terms of $4 i$ to get remaining base pairs of Q such as Qb21, Qb22, Qb31, Qb32, Qb41 and Qb42.
11. Click animation option. Thus, the 3D graphics shows the replication of PQ strand of the DNA.


### 3.3. GeoGebra Augmented Reality

In GeoGebra we have a special feature called 3D Augmented Reality (AR) where the projects created using the software in 3D Graphics can be viewed in natural environment. This has tremendous benefits when we use for learning purpose. For this we need phone or tab supporting this application.
App link : https://play.google.com/store/apps/details?id=com.google.ar.core
Below I have given some of my projects using GeoGebra.




## Chapter 4

## Challenges

Challenges that I faced during the fellowship:

- Researching deep mathematical concepts.
- Learing new commands and tools of GeoGebra software.
- Using LaTex to make presentations and report for the first time
- Exploring and understanding 3D augmented reality feature in GeoGebra
- Time management
- Revising chemistry concepts such as concentration and molarity


## Chapter 5

## Conclusion

In conclusion, being a part of the FOSSEE Summer Fellowship 2023 was a lifetime experience. It intrigued me to learn new concepts and analysing mathematical applications. Along with brushing up my understanding of deep mathematical concepts, I also learned a lot of valuable soft skills such as communicating with one's peers and mentors, and managing one's time efficiently. Additionally, I got a chance to learn about some new prospects of open source software such as Avagadro and Orca. Each of these newly imparted skills will be extremely useful in my professional career. I feel a sense of accomplishment and contentment knowing that my work in this fellowship would be beneficial to people who think maths is difficult will learn those concepts in deep using GeoGebra. I would like to express gratitude to all my mentors as well as my fellow interns who turned this fellowship into such a remarkable experience.

## Chapter 6

## Useful links

### 6.1. GeoGebra links.

Finding Definite integral, left Riemann sum, right Riemann sum and middle Riemann rule

## https://www.geogebra.org/classic/yntzdhqs

Matrix transformation of 3x3 matrix
https://www.geogebra.org/classic/uvcamcpa
Contour
https://www.geogebra.org/classic/rwpuextu
Spinning parametric curves
https://www.geogebra.org/classic/dh2dbyhk
Conic section
https://www.geogebra.org/classic/fms2kmfm
Rotation of conics
https://www.geogebra.org/classic/jpdbhfca
Revolution of conic surfaces
https://www.geogebra.org/classic/qa5dbakh
RC Circuit
https://www.geogebra.org/classic/ecmt8jhv
RL Circuit
https://www.geogebra.org/classic/nuwdbx6b
RLC Circuit.
https://www.geogebra.org/classic/brksztrk

Electric potential
https://www.geogebra.org/classic/tc3hdvc6
Electromagnetic wave
https://www.geogebra.org/classic/kvdaxrea
Butterfly curve
https://www.geogebra.org/classic/wsgaghu7
DNA model
https://www.geogebra.org/classic/fgsbpxdj
DNA replication model
https://www.geogebra.org/classic/ara4xxwr

### 6.2. Reference links.

https://youtu.be/WnPf5K2O5Ec?si=FGUHMI-a9Vxmrz o
https://youtu.be/sLjd3zDiPu8?si=G8jthyqSjXY8C0ST
https://youtu.be/P4NRkbqswb4?si=dbuAv8lN4JDClOx
https://youtu.be/q3FOwnoM0xY?si=Fqtehu2QpUIKGa0b
https://spoken-tutorial.org/tutorial-search/?
search foss=GeoGebra+5.04\&search language=English
https://spoken-tutorial.org/tutorial-search/?
search foss=Applications+of+GeoGebra\&search language=English

