FOSSEE Summer Fellowship Report

On

Mathematics using python

Submitted by

Vaishnavi Bisht

Birla Institute of

Applied Sciences

Email:

vnb0902@gmail.com

Under the guidance of

Prof. Prabhu Ramachandran

Department of Aerospace Engineering Indian Institute of Technology, Bombay

Mentors

Mr. Purusharth Saxena

Ms. Sharanya Achut

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I would like to express my special thanks of gratitude to my mentors: Ms. Sharanya Achut and Mr. Purusharth Saxena for their constant guidance, technical support and encouragement all along till the completion of this fellowship.

I respect and thank Prof. Prabhu Ramachandran, for providing detailed reviews and astute suggestions that helped me a lot in improving my lecture notes and animations. I am extremely thankful to him and my mentors for providing such a nice support and guidance.

I would also like to thank my other fellows for their valuable feedbacks and active responses to my queries.

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Introduction

1.1 FOSSEE Animations and the Summer Fellowship

What is FOSSEE?

FOSSEE (Free/Libre and Open Source Software for Education) project is part of the National Mission on Education through Information and Communication Technology (ICT), Ministry of Human Resource Development (MHRD), Government of India. FOSSEE project promotes the use of FLOSS tools to improve the quality of education in the country and aims to reduce dependency on proprietary software in educational institutions.

What is FOSSEE Animations?

FOSSEE animations is a library of community-curated open source animations on mathematical topics to promote animation-based learning. All the animations are made using python open source libraries such as manim or mayavi.

For this fellowship I was assigned with the task of creating mathematical lecture notes and animations for the topic- Approximations and Optimizations. The main topic was divided into 5 sub topics and for each subtopic I had to prepare one lecture note and a minimum of 3 animations per lecture note using manim.

Manim is an animation engine for explanatory math videos created by Grant Sanderson. It's used to create precise animations programmatically. It is composed of 5 parts:

- Python: it is used to write commands for creating animations.
- Latex: it is used for writing texts and formulas
- Cairo: it is the program that makes the figures
- ffmpeg: brings the frames together to form the video
- Sox: it is a cross-platform audio editing software

1.2 Topic for the Fellowship – Approximations and Optimization

"All models are approximations. Essentially, all models are wrong, but some are useful. However, the approximate nature of the model must always be borne in mind."-George Edward Pelham Box

In the real world, there is often not an equation, but just data that describes a situation, and approximation is one of the best ways that can be used to deal with any given collection of numerical data.

This topic is divided into 5 subtopics:

- Critical Points
- The Second Derivative Test
- Tangent Plane Approximations
- Total Differential
- Lagrange Multipliers

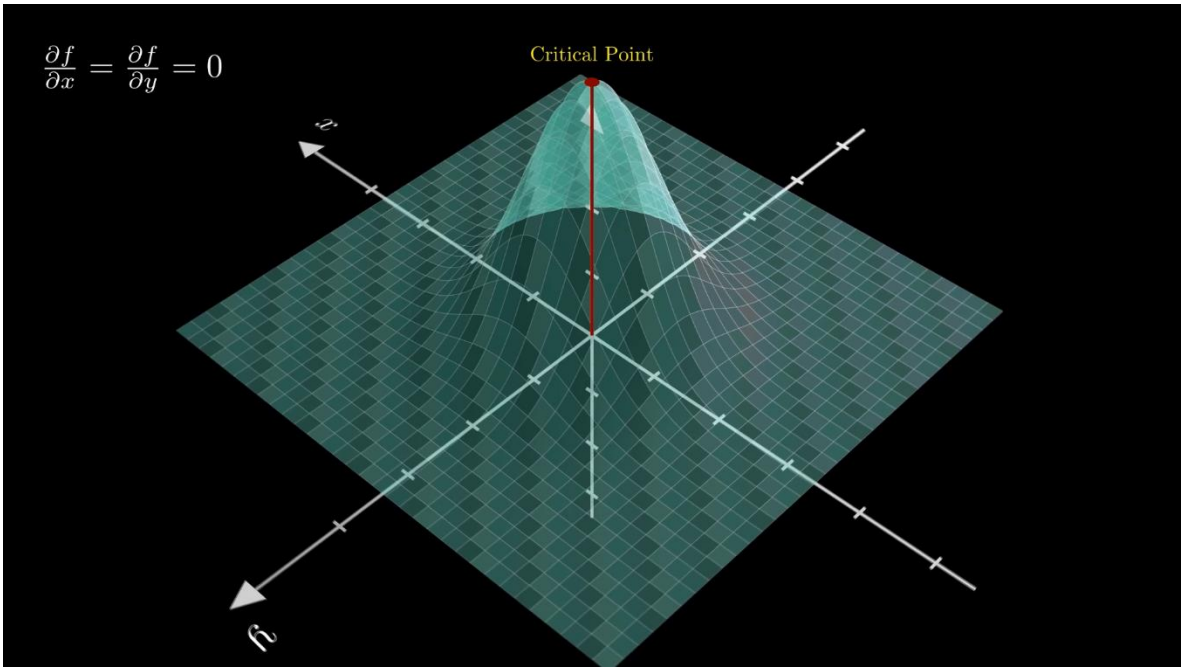
GitHub: https://github.com/FOSSEE/FSF-mathematics-python-code-archive/tree/master/FSF-2020/calculus-of-several-variables/approximations-and-optimizations

Critical Points

Link for the subtopic : https://math.animations.fossee.in/contents/calculus-of-several-variables/approximations-and-optimization/critical-points

Critical Points of a multivariable function are points in the function around which the rate of change is drastically altered. The following points are expounded in the lecture note for the same.

- Use of first-order partial derivatives in determining the critical point.
- What happens when only $f_x = 0$ is considered for finding critical points
- Proof of first derivative test
- Extrema occur only at critical points but not all critical points are extrema
- Types of critical points



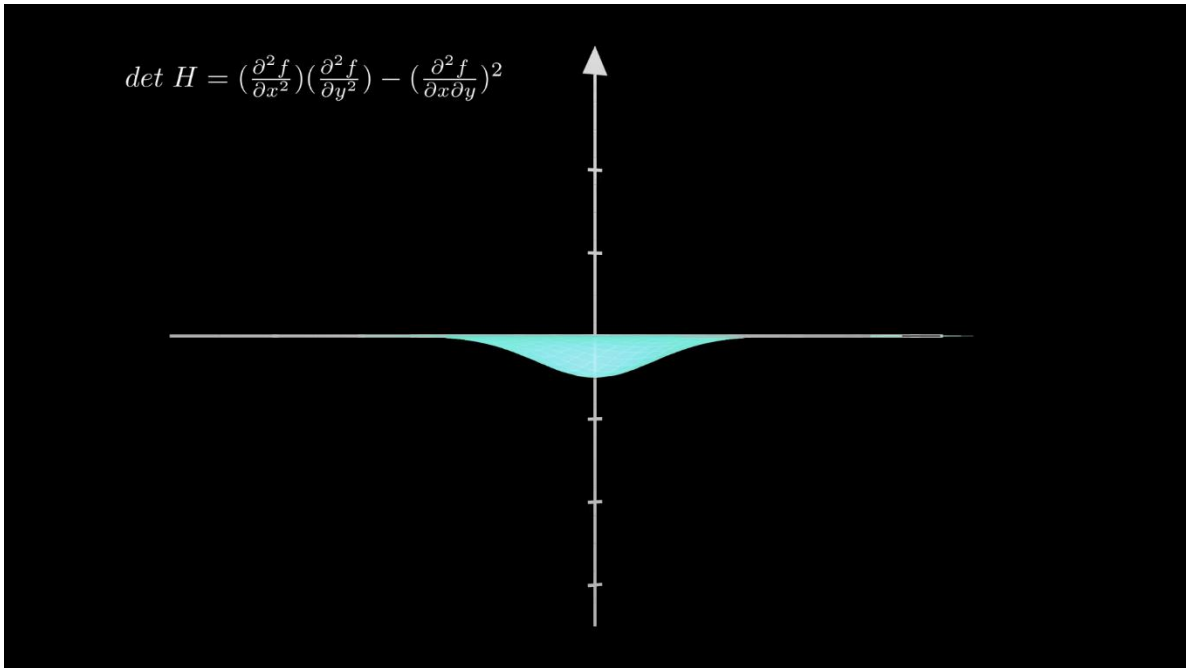
GitHub: https://github.com/FOSSEE/FSF-mathematics-python-code-archive/tree/master/FSF-2020/calculus-of-several-variables/approximations-and-optimizations/Critical-Points

The Second Derivative Test

Link for the subtopic : https://math.animations.fossee.in/contents/calculus-of-several-variables/approximations-and-optimization/the-second-derivative-test

Since **the first derivative test** is only limited to finding the critical points of a function, to analyze the change in the value of the function (either increasing or decreasing) or to analyze the concavity of the function at the critical point, **the second derivative test** is employed. The following points are expounded in the lecture note for the same.

- Use of second-order partial derivatives in second derivative test
- Hessian Matric and its type
- Need of finding the determinant of Hessian Matrix
- Contour Diagrams of different types of critical points



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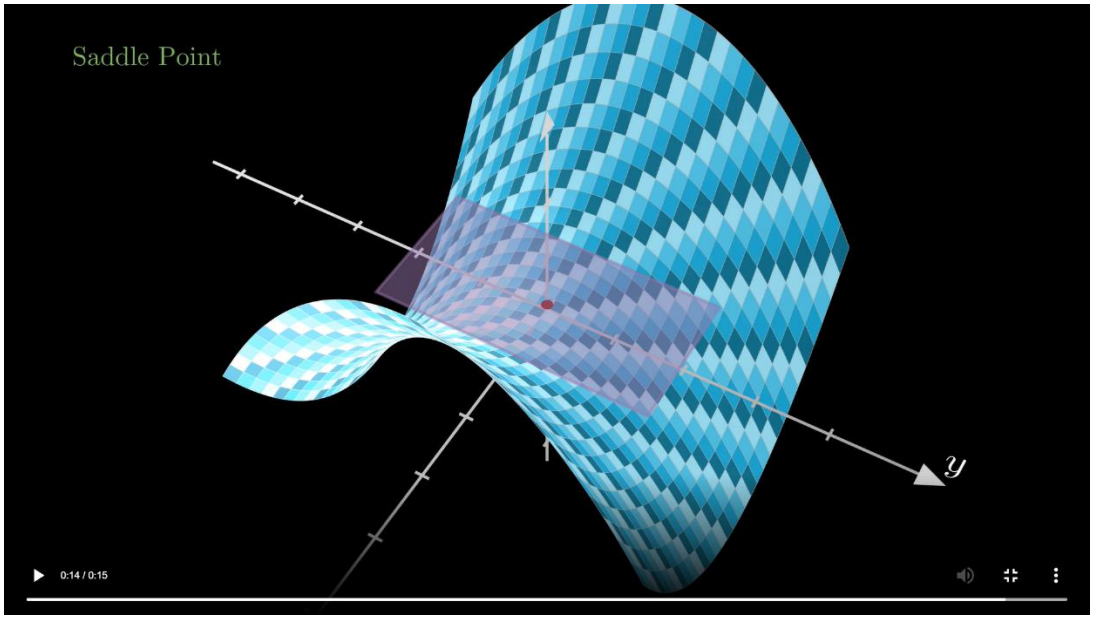
Tangent Plane Approximations

Link for the subtopic : https://math.animations.fossee.in/contents/calculus-of-several-variables/approximations-and-optimization/tangent-plane-approximations

Consider a smooth surface f described by a differentiable function f and focus on some point (x, y) on the surface. When zooming on that point, the surface would appear more and more like a plane. This plane is called the tangent plane. This tangent plane is used to approximate the values of the function at the points that lie near to (x, y) on the surface of the function. The following points are expounded in the lecture note for the same.

- When zoomed on the surface near critical point, the surface appears as a plane.
- Derivation of the tangent plane approximation
- Approximating value of the function at point(x, y) that lie near point (x_0, y_0)
- What is a tangent plane



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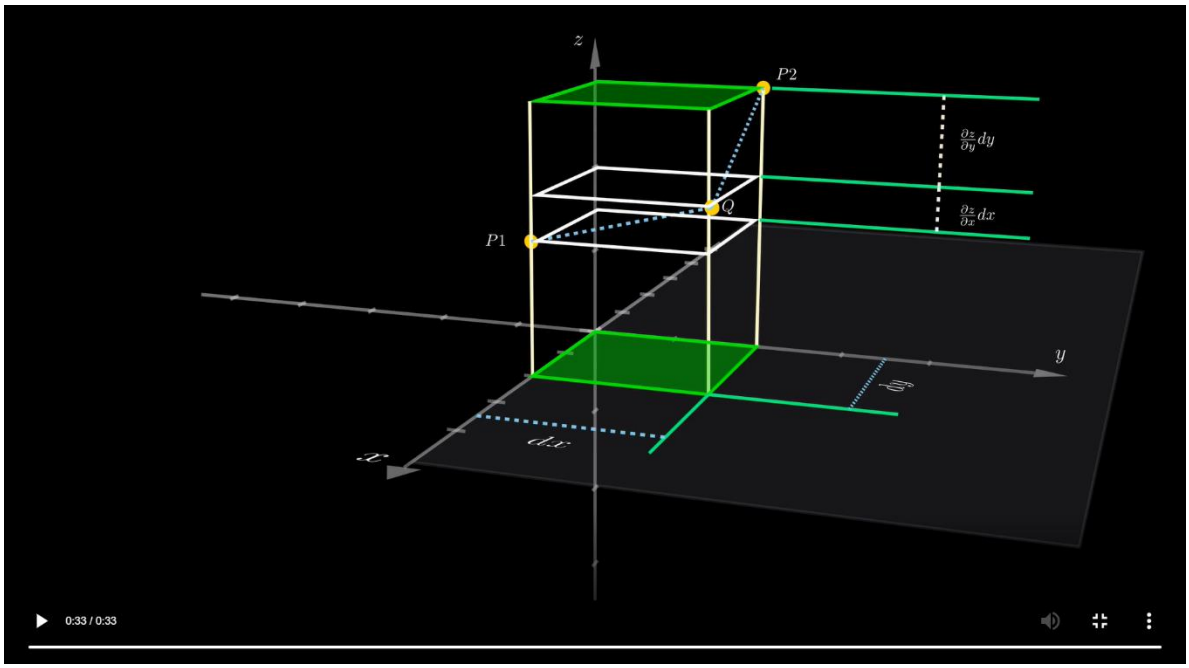
Total Differential

Link for the subtopic : https://math.animations.fossee.in/contents/calculus-of-several-variables/approximations-and-optimization/total-differential

Total differential used for finding the total change in the value of output for a small change in the values of the input variables. The following points are expounded in the lecture note for the same.

- What is dz
- Approximation using total differentials
- Determining the change in the value of the function using total differential when moving from point P_1 to P_2 on the surface of the function.



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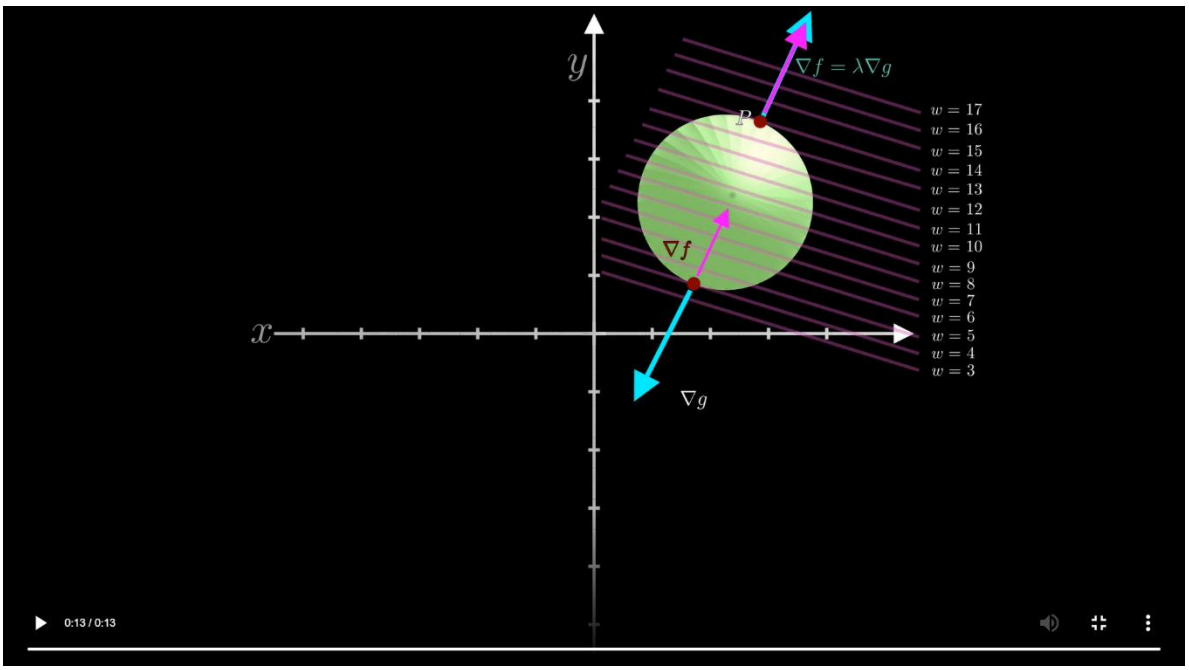
Lagrange Multipliers

Link for the subtopic : https://math.animations.fossee.in/contents/calculus-of-several-variables/approximations-and-optimization/lagrange-multipliers

Lagrange multipliers is a method for finding extrema of a differentiable function of several variables subject to a given constraint (the equation(s) that may describe the boundary of a region). The method of Lagrange multipliers is useful in solving constrained-optimization problems. The following points are expounded in the lecture note for the same.

- Using Lagrange Multiplier to solve constrained optimization problem
- Lagrangian Function



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Conclusion

I learned a lot during this fellowship. I got to study a lot of new mathematical topics in-depth and I learned how to write educational, easy-to-understand and concise lecture notes. Thanks to the tutorial videos created by my mentors, I learned how to create 3D mathematical animations using manim.

Due to the exhaustive reviewing process, my approach of writing lecture notes as well as my animations gradually improved a lot during this fellowship.

At the end, I would like to thank and appreciate everyone who made my fellowship a superb learning and memorable experience.