FOSSEE Summer Fellowship Report

On

Mathematics Using Python

Submitted by

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Acknowledgement

I, the summer intern of the **FOSSEE - Mathematics using Python** am overwhelmed in all humbleness and gratefulness to acknowledge my sincere gratitude to all those who have helped me put my ideas to perfection and have assigned tasks well above the level of simplicity and into something concrete and unique. I wholeheartedly thank **Prof. Prabhu Ramachandran** for having faith in me, selecting me to be a part of his valuable project, and for continually motivating me to do better. I thank **Mr. Purusharth Saxena** and **Ms. Sharanya Achut** for providing me with the opportunity to work on this project, and for their valuable suggestions. They were and are always there to show me the right track when needed help. With the help of their brilliant guidance and encouragement, I was able to complete my tasks correctly and was up to the mark in all the assigned tasks. During the process, I got a chance to see the stronger side of my technical and non-technical aspects and strengthen my concepts. Last but not the least, I sincerely thank all other colleagues working in different projects under **Prof. Prabhu Ramachandran** for helping me evolve better with their critical advice.

Contents

Chapter 1

Introduction

1.1 FOSSEE Animations and the Summer Fellowship

The Free/Libre and Open Source Software for Education (FOSSEE) project promotes the use of FLOSS tools in academia and research, and is also part of the National Mission on Education through Information and Communication Technology (ICT), Ministry of Human Resource Development (MHRD), Government of India. FOSSEE Animations is a library of community-curated animations on Science and Math using open source tools.

Work assigned: Create lecture notes and animations for various topics in Advanced Higher Mathematics using open source tools for ease in visually understanding various concepts.

Visualizing and simplifying complex topics to make them easily understandable is an appreciable contribution not only to the end-user in the Science and Math community, but to the writer as well.

Tools used for the animations include: Manim, MatplotLib, Python, LaTeX.

1.2 Topic for the Fellowship - Geometry of Planes and Curves

Multivariable calculus requires a geometrical understanding and interpretation of various concepts, from basics, such as the equations of planes and lines, to slightly advanced topics such as curvature and torsion.

A personal opinion on this topic: the stronger and clearer one's fundamentals are, the higher their reach in advancing in its depth can be. To be able to visualize various concepts which we see in everyday life, but from a mathematical perspective: is a luxury not everyone is blessed with.

GitHub: Link

Chapter 2

Space Curves - an Introduction to Coordinates in 3D

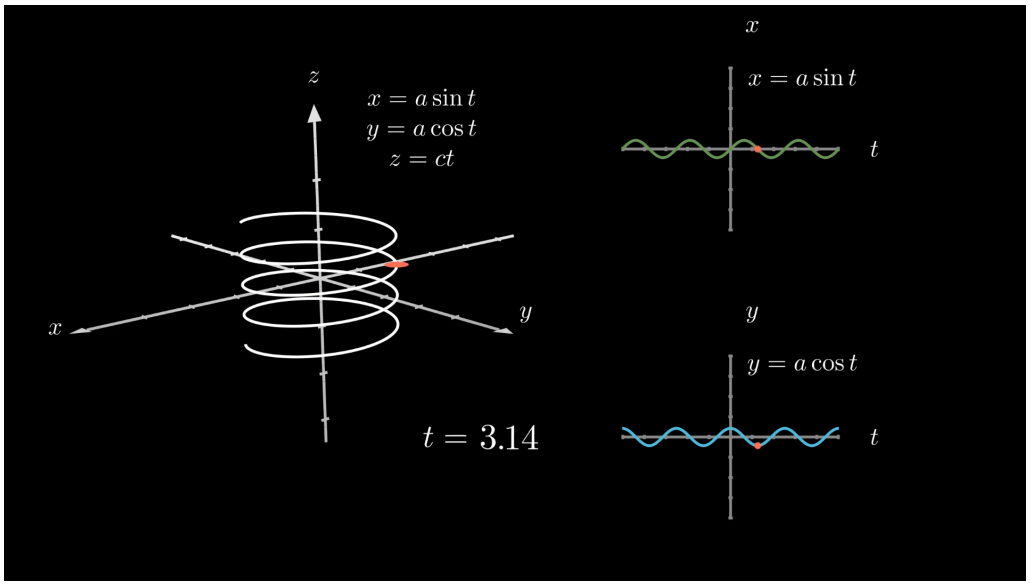
Lecture Notes: Link **GitHub:** Link

Through the course of these notes, the following topics have been covered:

- An introduction to the 3D framework and right-hand convention.
- When do parametric / space curves come into play?
- Why are parametric curves used for calculating and understanding instantaneous aspects along a curve?
- Why should sketching be done by eliminating the parameter?

Number of animations created: 4

The screenshot of an animation depicting the representation of helix as a vector valued function of a parameter can be seen attached below.



Chapter 3

Equations of Planes and Lines

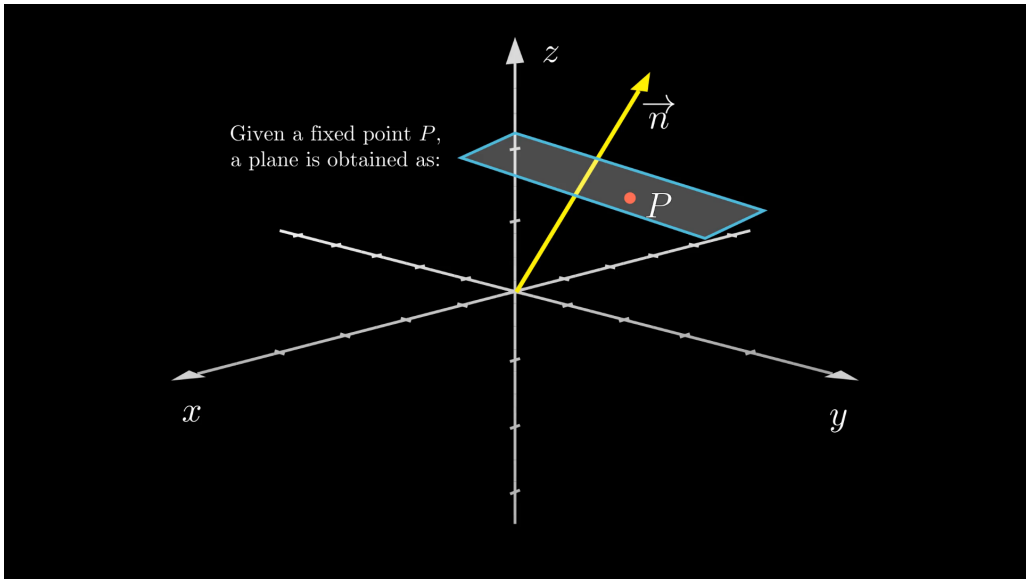
Lecture Notes: Link **GitHub:** Link

Through the course of these notes, the following topics have been covered:

- Why both: the point and the normal are required to find the equation of a plane?
- Inter-conversion between the vector, scalar and intercept forms of a plane.
- Similarly, why both: a point and the direction are required to find the equation of a line?
- Special case of constants in the scalar form of line.
- Intersection of lines and planes.

Number of animations created: 5

The screenshot of an animation depicting the point-normal form for the equation of a plane can be seen attached below.



Chapter 4

General Parametric Curves

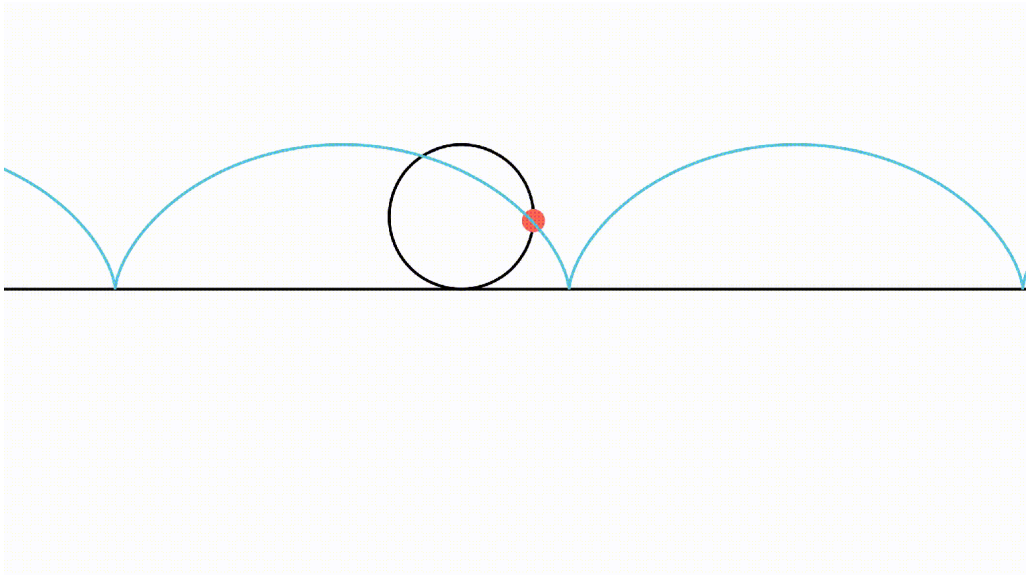
Lecture Notes: Link **GitHub:** Link

Through the course of these notes, the following topics have been covered:

- Parametric equations of an origin-centred, as well as arbitrary point-centred circle.
- Derivation for the parametric equations of a cycloid.
- The famous Brachistochrone and Tautochrone problems.
- Parametric equations of a helix.
- How is a helix bounded by the corresponding cylinder?

Number of animations created: 4

The screenshot of an animation depicting the generation of a cycloid can be seen attached below.



Chapter 5

Velocity and Differentiability

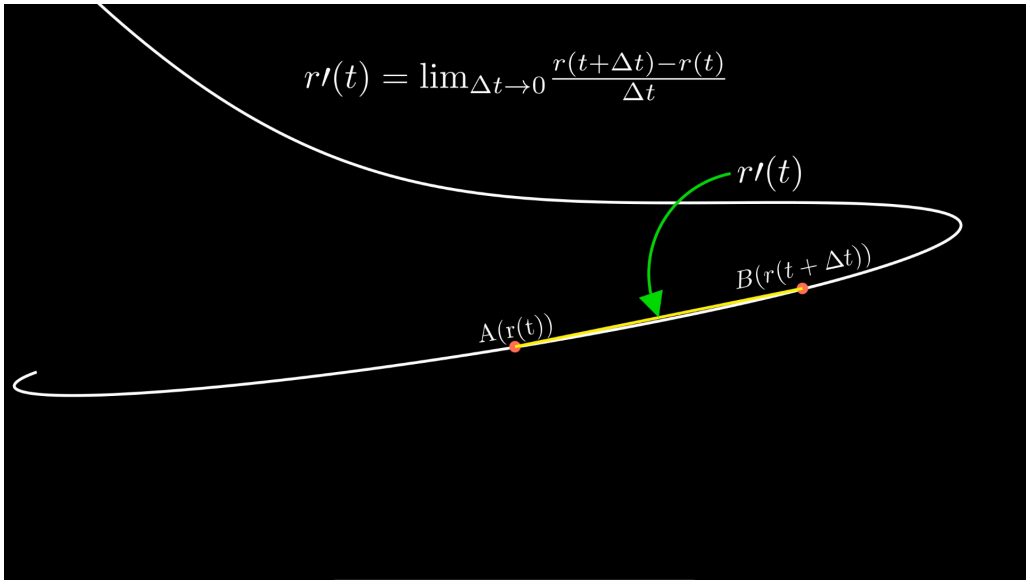
Lecture Notes: Link **GitHub:** Link

Through the course of these notes, the following topics have been covered:

- Understanding and visualization of the concepts of differentiability and continuity.
- Discussing whether the velocity of a moving point can be obtained for any space curve.
- Introduction to smooth curves.
- A brief on tangent lines to space curves.
- Discussing the concepts of peed and acceleration, from a calculus point-of-view.

Number of animations created: 3

The screenshot of an animation depicting the interpretation of a derivative in 3D can be seen attached below.



Chapter 6

Arc Length and Curvature

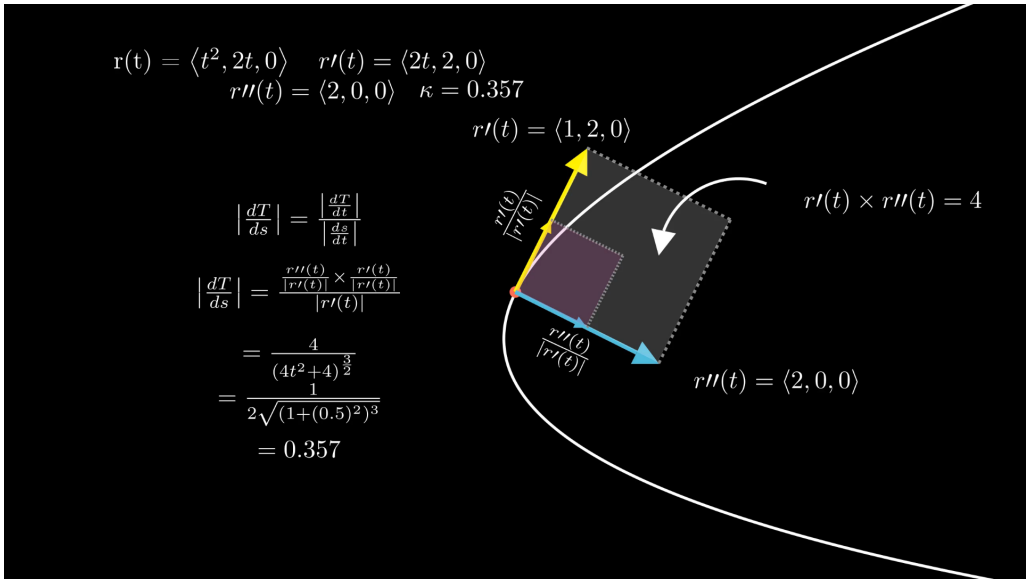
Lecture Notes: Link **GitHub:** Link

Through the course of these notes, the following topics have been covered:

- Understanding and visualizing the concept of arc length.
- Computing the arc length for different types of curves.
- Studying curvature: from a high-level view to an in-depth understanding.
- Derivation of the curvature formula and its geometrical interpretation.
- Being able to intuitively understand and acknowledge the concepts pertaining to curvature.

Number of animations created: 5

The screenshot of an animation depicting the geometrical interpretation of curvature at a given point can be seen attached below.



Chapter 7

The TNB Frame and Serret-Frenet Formulae

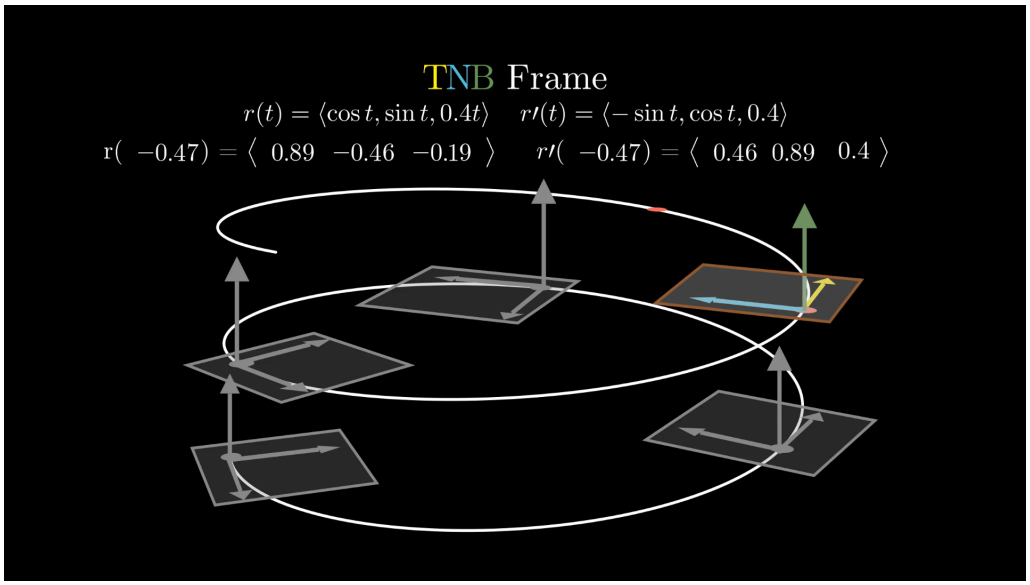
Lecture Notes: Link **GitHub:** Link

Through the course of these notes, the following topics have been covered:

- Understanding the basics of unit tangent, normal, and binormal vectors.
- Plane prescribed by the unit tangent and normal vectors at an arbitrary point on a curve.
- Creation and visualization of a TNB frame.
- Derivation, visualization and geometrical interpretation of the Serret-Frenet formulae.
- A study on the effect of torsion and curvature on the unit normal vector.

Number of animations created: 7

The screenshot of an animation depicting the geometrical interpretation of curvature at a given point can be seen attached below.



Chapter 8

Conclusion

Visiting a few topics studied previously and learning them with a bright new light and understanding was extremely joyful an experience. My perspective towards mathematics has drastically changed from "maximizing number of computations per unit time" to "the beauty of conceptual visualization", and I am forever grateful for this opportunity.

One of the challenges faced was to visualize torsion, while writing the notes for the Serret-Frenet formulae. My mentors spent a great amount of time: not just once, but time and again, to help me grasp it clearly.

My experience with FOSSEE was crucial as well as extraordinarily rewarding in the development of my mathematical knowledge. I will definitely carry the lessons and skills learned to my next position, and hope to have the honor of working under **Prof. Prabhu Ramachandran** once more in the near future.