FOSSEE Summer Fellowship Report

On

Mathematics using Python

Submitted by

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Acknowledgment

In doing this project, I had the privilege of working with a wonderful group of people, my fellow contributors, each of whom had a unique approach to animating and visualising math, and from each of whom I was able to pick up a lesson or two.

I would like to thank the team at FOSSEE at IIT-B for this opportunity, and for promoting open-source projects associated with STEM education. As a student, this experience marks the beginning of my (rather small) contribution to the open-source community, and I am indebted to FOSSEE for giving me this start.

I would like to extend my gratitude towards Prof Prabhu Ramachandran, under whose guidance I was able to do this project. I am both grateful to and greatly inspired by Grant Sanderson, Prof Prabhu Ramachandran, and Michael Droettboom (and everyone else who contributed), who are credited with the development of Manim, MayaVi, and Matplotlib respectively. How lucky we are today, to have access to such tools that can assist and enhance our learning of mathematics and science.

Last, but not the least, I would like to thank my mentors, Sharanya and Purusharth, who've spent hours helping me with my work. I have learnt a lot from them. Without their support, guidance, advice, and encouragement, this project would have been impossible.

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Introduction

1.1 FOSSEE Animations and the Summer Fellowship

The **FOSSEE** (Free/Libre and Open Source Software for Education) project promotes the use of FLOSS tools in academia and research, and is part of the National Mission on Education through Information and Communication Technology (ICT) and Ministry of Human Resource Development (MHRD), Government of India. FOSSEE has fostered projects in Scilab, Python, OSDAG, and OpenModelica, among others.

FOSSEE Animations is a library of community-curated animations on various topics in mathematics and science, made using open-source tools. The **Mathematics using Python** project required fellows to create lecture notes on topics in advanced mathematics. Apart from writing notes for a particular concept, our work involved making relevant illustrations and animations that would help simplify the concept and help the reader visualise it. Learning which focuses on understanding and intuition, rather than memorisation and the mere application of formulae, is extremely valuable and useful. Our project was driven by this philosophy.

1.1.1 Tools used for animation and visualisation

- Manim - for 2D and 3D animation
- MayaVi - to create 3D vector field animation
- Matplotlib - for surfaceplots, streamplots, etc.
- SciPy - for numerical and scientific computing

1.2 Topic for the Fellowship - Div, Grad, Curl, and all that

Oftentimes, concepts and ideas in mathematics are better appreciated when their physical significance becomes apparent. This is certainly true of vector calculus, particularly with regard to the operators divergence, curl, and gradient. The geometrical interpretation and visualisation of these ideas makes it a deeply insightful topic to explore. Div, curl, grad, and the theorems associated with them are extremely popular for their applications in engineering, physics, economics, biology and life sciences, etc. Vector calculus forms a foundation for topics like differential geometry, topology, and tensor calculus, among others.

GitHub: https://github.com/FOSSEE/FSF-mathematics-python-code-archive/tree/master/FS
2020/calculus-of-several-variables/div-curl-grad-and-all-that

Gradient

Lecture Notes: https://math.animations.fossee.in/contents/calculus-of-several-variables/integrals-of-multivariable-functions/gradient

GitHub: https://github.com/FOSSEE/FSF-mathematics-python-code-archive/tree/master/FSF-2020/calculus-of-several-variables/div-curl-grad-and-all-that/gradient

In the notes on **Gradient**, we introduce the gradient as an analogue of the derivative in higher dimensions, and then proceed to cover the properties and applications of the operator. A section is dedicated to discussing the intuition behind the formula used to compute gradient.

Number of animations/illustrations - 8

Figure 2.1: A still from the animation explaining the grad formula. Created using manim.

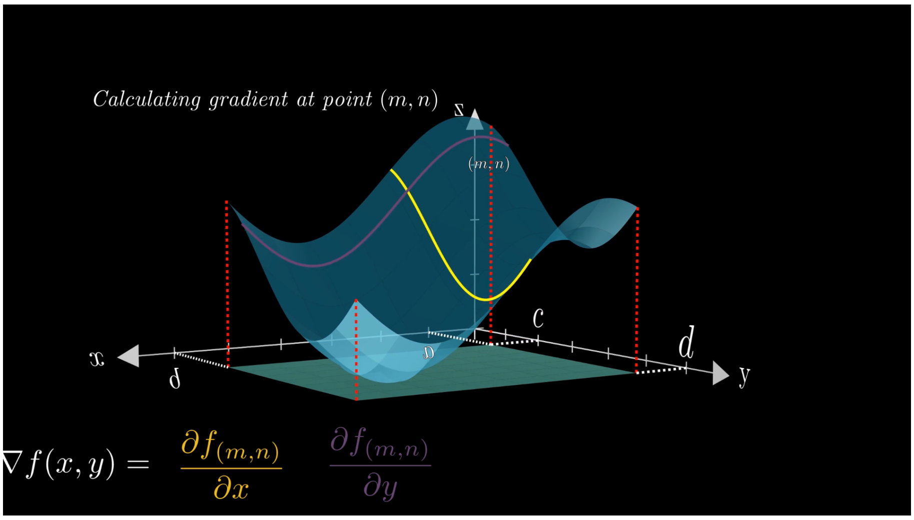
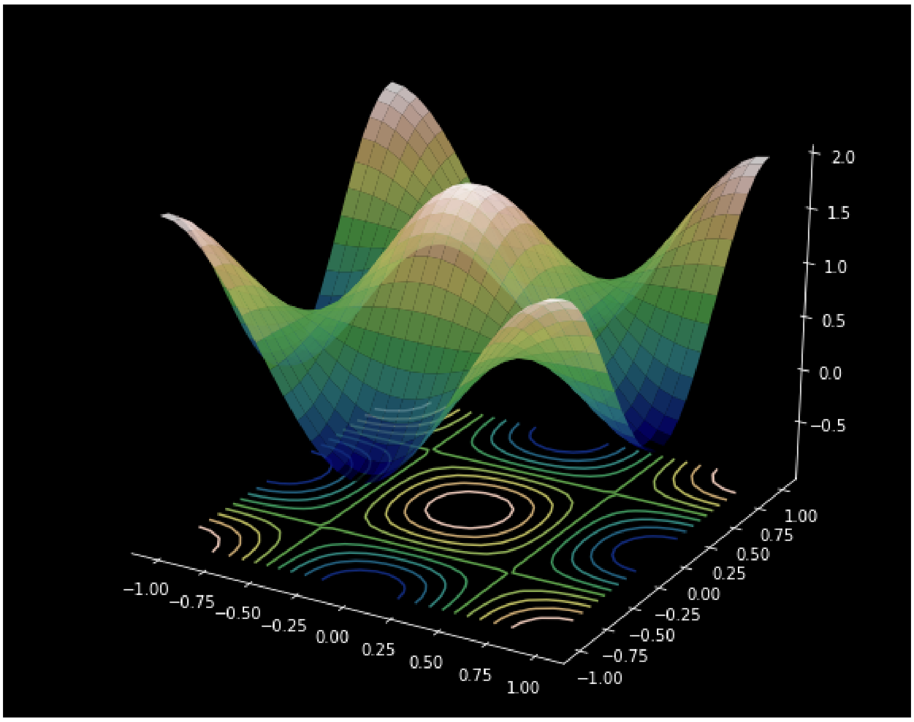


Figure 2.2: A figure that depicts a surface and its contour plot. Created using matplotlib.



Curl and Div

Lecture Notes: https://math.animations.fossee.in/contents/calculus-of-several-variables/div-grad-curl-and-all-that/curl-and-divergence

GitHub: https://github.com/FOSSEE/FSF-mathematics-python-code-archive/tree/master/FSF-2020/calculus-of-several-variables/div-curl-grad-and-all-that/curl-and-div

The notes on **Curl and Divergence** start with visualising div and curl with the help of fluid flow. We move on to discussing the intuition behind the formulae, and then draw the reader's attention to the subtle difference between microscopic and macroscopic curl. The last part of the notes deal with the formal definitions and applications of the operators.

Number of animations/illustrations - 12

Figure 3.1: A still from the animation explaining the formal definitions of div and curl. Created using manim.

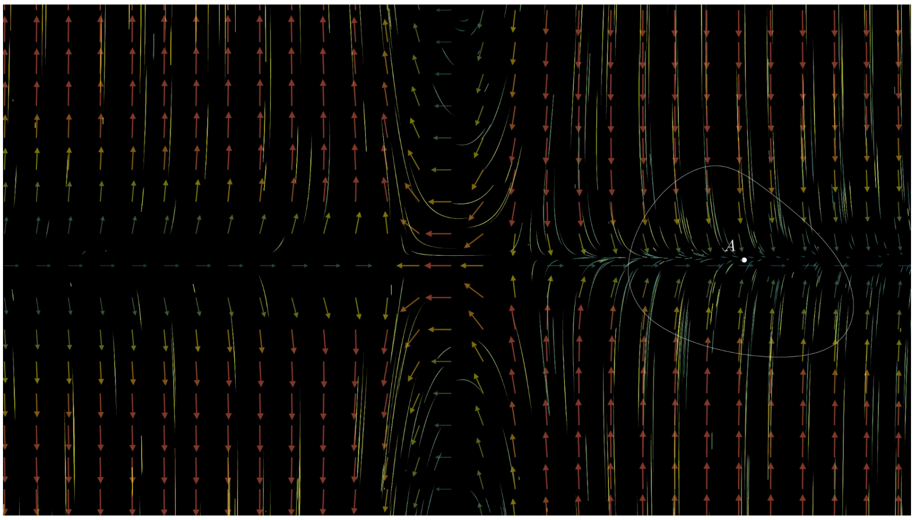
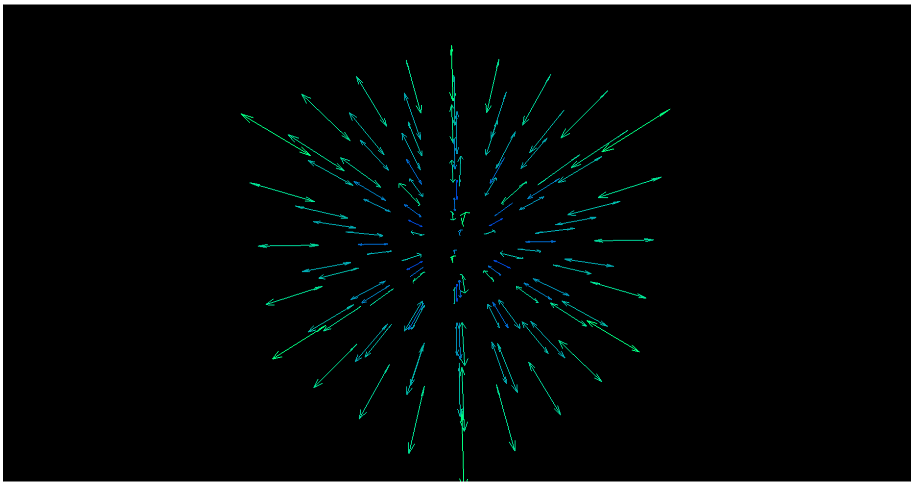


Figure 3.2: A still from the animation that shows a three dimensional vector field. Created using mayavi.



Conservative Fields, Path Independence, Gradient Fields

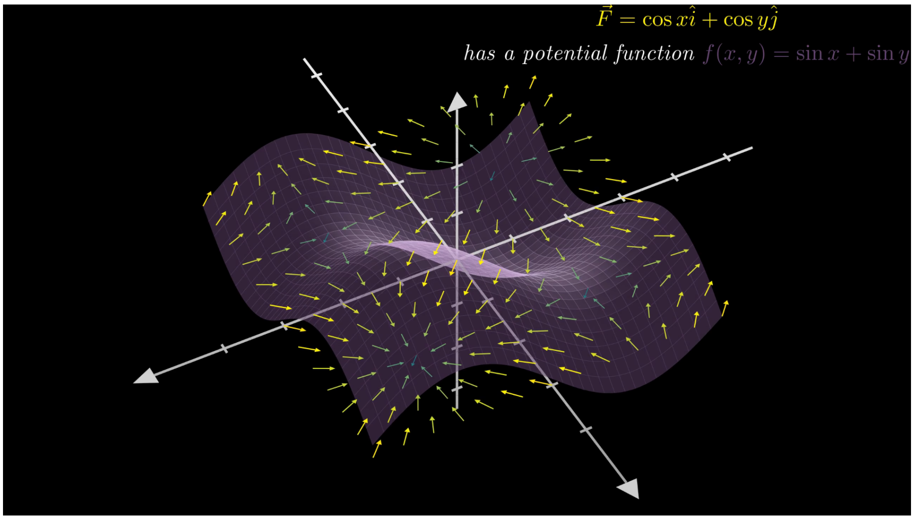
Lecture Notes: https://math.animations.fossee.in/contents/calculus-of-several-variables/div-grad-curl-and-all-that/conservative-fields,-path-independance,-gradient-fields

GitHub: https://github.com/FOSSEE/FSF-mathematics-python-code-archive/tree/master/FSF-2020/calculus-of-several-variables/div-curl-grad-and-all-that/conservative-fields

In the notes on **Conservative Fields, Path Independence, Gradient Fields** we cover conservative fields, path independent and path dependent vector fields, gradient fields, and potential functions. The section is motivated using the example of the gravitational field.

Number of animations/illustrations - 6

Figure 4.1: A still from the animation explaining the formal definitions of div and curl. Created using manim.



$\vec{F}$ is conservative $\implies$ curl $\vec{F} = 0$

Lecture Notes: https://math.animations.fossee.in/contents/calculus-of-several-variables/div-grad-curl-and-all-that/vec-f-is-conservative-implies-curl-f=0

GitHub: https://github.com/FOSSEE/FSF-mathematics-python-code-archive/tree/master/FSF-2020/calculus-of-several-variables/div-curl-grad-and-all-that/vec-f-is-conservative-implies-curl-f-is-0

The notes on $\vec{F}$ **is conservative** $\implies$ **curl** $\vec{F} = 0$ cover the theorem and its proof, introducing it using the example of the vortex vector field. The section ends after a brief note on simply connected regions.

Number of animations/illustrations - 6

Figure 5.1: A still from the animation that explains curl using the popular paddle-wheel analogy . Created using manim.

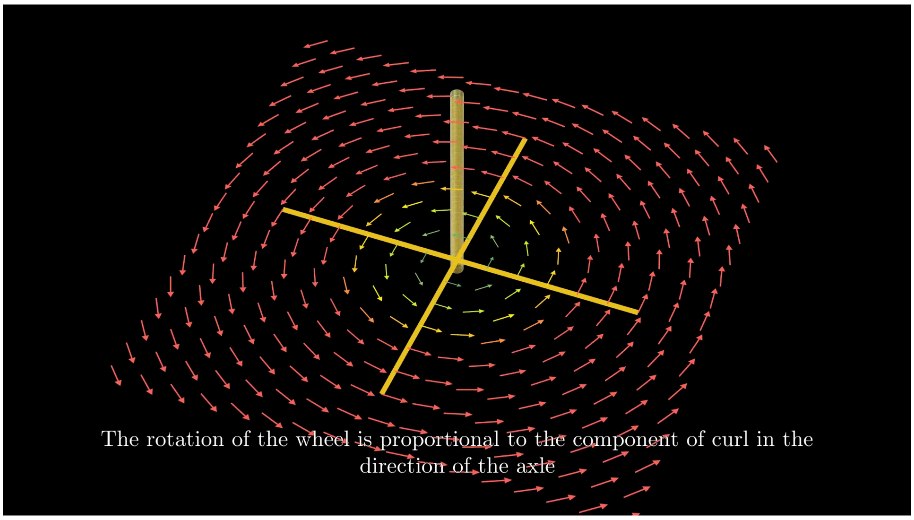
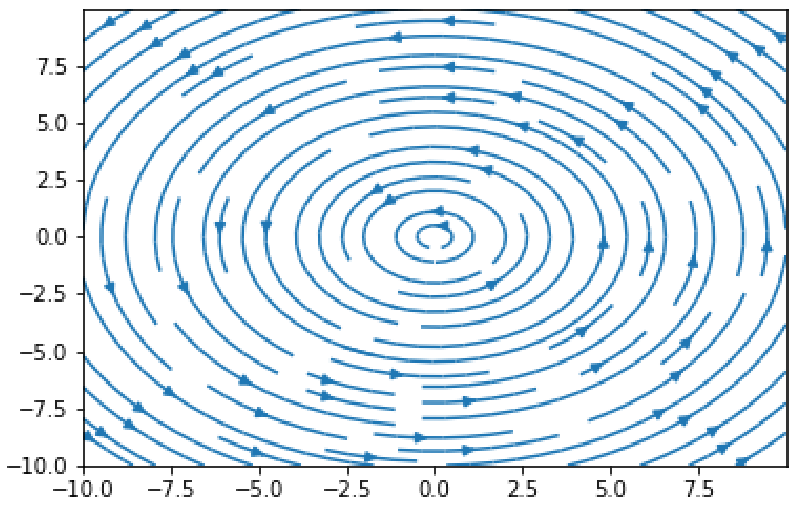


Figure 5.2: A streamplot of the vortex vector field. Created using matplotlib.



Green's Theorem

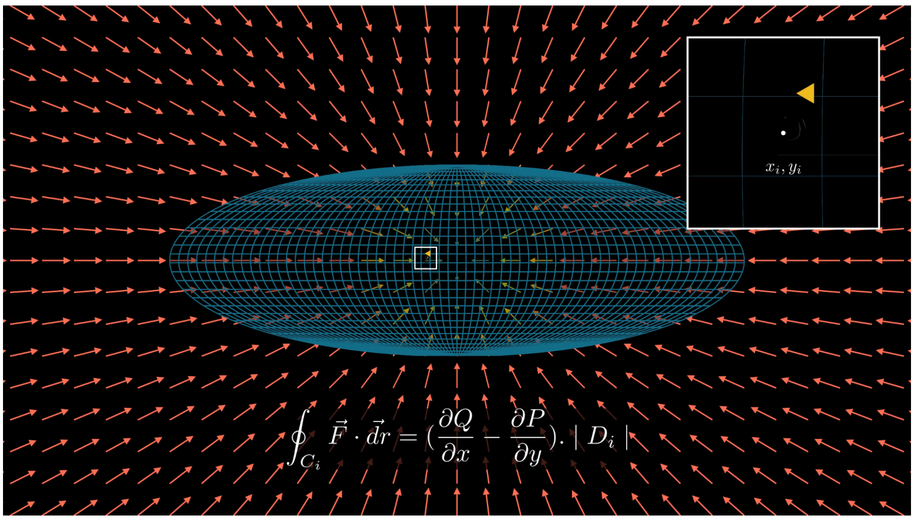
Lecture Notes: https://math.animations.fossee.in/contents/calculus-of-several-variables/div-grad-curl-and-all-that/green's-theorem

GitHub: https://github.com/FOSSEE/FSF-mathematics-python-code-archive/tree/master/FSF-2020/calculus-of-several-variables/div-curl-grad-and-all-that/greens-theorem
A brief summary of the concepts covered in the notes on **Green's Theorem**:

In the notes on **Green's Theorem** we focus predominantly on understanding the proof of the theorem by visualising it. A section on computing areas using Green's Theorem is also included.

Number of animations/illustrations - 4

Figure 6.1: A still from the animation on the proof of the theorem . Created using manim.



Conclusion

"If you can't explain it simply, you don't understand it well enough."
-Albert Einstein

Developing an intuition for concepts in mathematics is no easy task, let alone articulating the insight you've gained to a varied audience. Being offered this fellowship gave me an opportunity to learn vector calculus the right way. It has helped me grow as a student of mathematics. The process of creating lecture notes taught me how valuable clarity in thought is. One of the early challenges I faced was with installing software. However, with the help of my mentors, we got it up and running in no time. I am especially grateful for the reviews we had with Prof Ramachandran. I was forced to reconsider my approach towards the subject several times, and came out having improved my understanding as well as my ability to articulate each time. All in all, it was a wonderful learning experience for me.