FOSSEE Summer Fellowship Report

On

Mathematics Using Python

Submitted by

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July 22, 2020

Acknowledgment

It is a pleasure to acknowledge the help and support that has gone to in making this successful project. I would like to thank the entire FOSSEE Team at IIT Bombay, for providing me with this wonderful opportunity.

I would like to take this opportunity to express my greatest gratitude to my mentors, **Ms. Sharanya Achut** and **Mr. Purusharath Saxena** for being a constant support, and helping me to push my limits. They were consistently there at whatever point I required assistance and furnished me with their valuable guidance and consolation, so I'm in the correct track. This in turn has helped me to put my plans flawlessly and make concrete and one of a kind notes.

I wholeheartedly thank **Prof. Prabhu Ramachandran** for his guidance and invaluable suggestions. Last but not the least, I'd like to truly appreciate all other fellow's efforts for helping out, with the problems I had faced, and making this fellowship, a memorable and enjoyable experience.

Contents

1 Introduction **3**
1.1 FOSSEE Animations and the Summer Fellowship 3
1.2 Topic for the Fellowship - Multivariable Functions and Partial Derivatives 4

2 Scalar Functions **5**

3 Multivariable Functions **6**

4 Multivariable Limits and Continuity **7**

5 Partial Derivatives **8**

6 Directional Derivatives **9**

7 Conclusion **10**

Introduction

1.1 FOSSEE Animations and the Summer Fellowship

The Free/Libre and Open Source Software for Education (**FOSSEE**) project promotes the use of FLOSS tools in academia and research. The FOSSEE project is funded and is a part of the National Mission on Education through Information and Communication Technology (ICT), Ministry of Human Resource Development (MHRD), Government of India.

FOSSEE Animations is a library of open-source community-curated animations on Science and Math, made using open-source libraries like Manim, Mayavi, Blender and VPython.

The assigned work was to create lecture notes and animations for various subtopics in Multivariable Functions and Partial Derivatives. The thought process was to offer visualisation to these topics, making these notes informative and easy to understand.

In the scientific community, it's an invaluable contribution by making various complex mathematical topics approachable through visualisation.

These animations were majorly made using Manim, while some of them made using Octave version 5.2.

1.2 Topic for the Fellowship - Multivariable Functions and Partial Derivatives

"The process of selecting a candidate function or hypothesis to model a world is what great geniuses of science is remembered for." - unknown

Our physical world is multidimensional, having a lot of the scientific areas rich in multivariable mathematics. Its application to problem solving in the multidimensional world resulted in the need for generalization to include functions of more than one variable and multivariable calculus.

The subtopics assigned in Multivariable Functions and Partial Derivatives were Scalar Functions, Multivariable Functions, Multivariable Limits and Continuity, Partial Derivatives and Directional Derivatives.

GitHub: **Multivariable Functions and Paritial Derivatives**

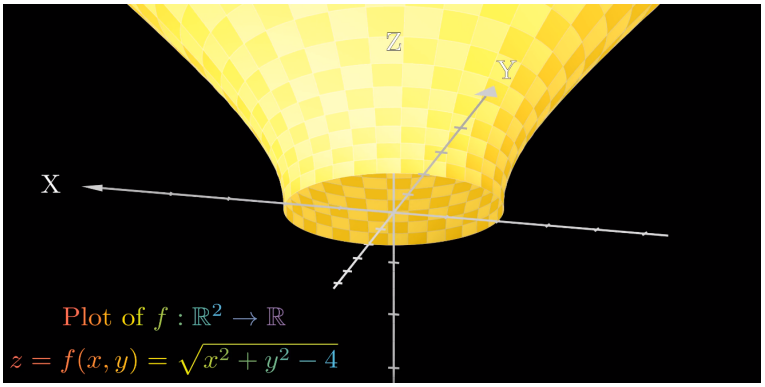
Scalar Functions

Lecture Notes: Scalar Functions

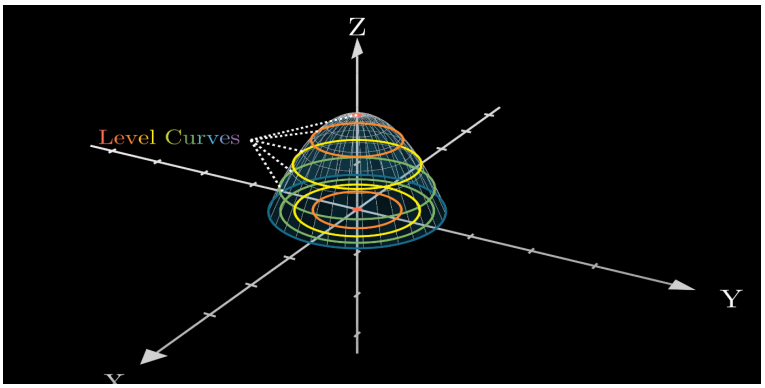
Throughout the lecture notes, the following topics have been covered:

- Introduction to scalar functions
- Domain and range of a scalar function
- Concept on the mapping of scalar function from n-dimensional space to one-dimensional real-valued space
- What are Level sets?
- Types of level sets: Level curves and Level surfaces.

The following animation's screenshot shows the plot of function $z = f(x, y)$.



The following animation's screenshot displays the level curves.



GitHub: Scalar Functions

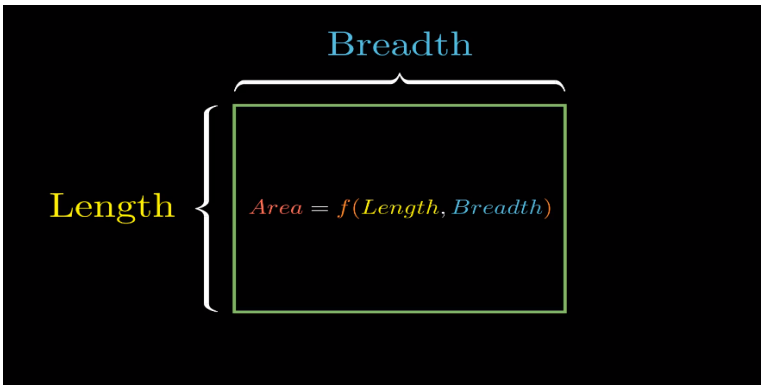
Multivariable Functions

Lecture Notes: Multivariable Functions

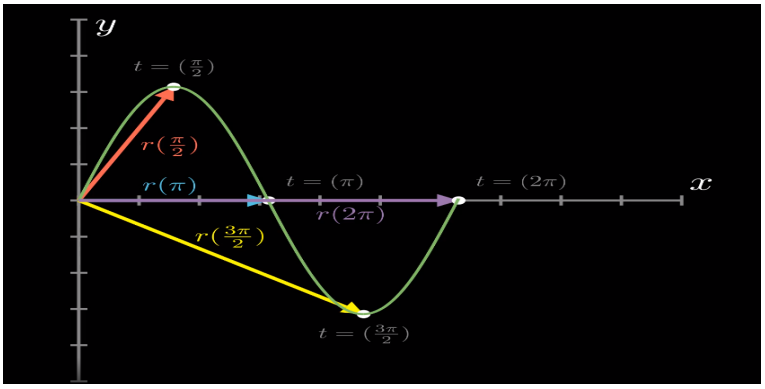
Throughout the lecture notes, the following topics have been covered:

- Introduction to multivariable functions and it's types
- Concept of a point as the input of a multivariable function in n-dimensional space
- Introduction to vector-valued function
- Differentiation of a vector-valued function

The following animation's screenshot represent the area of rectangle as function of it's length and it's breadth.



The following animation's screenshot shows an example of a vector-value function.



GitHub: Multivariable Functions

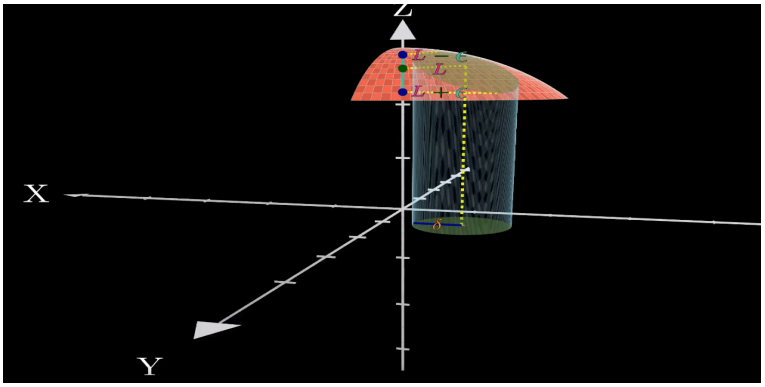
Multivariable Limits and Continuity

Lecture Notes: Multivariable Limits and Continuity

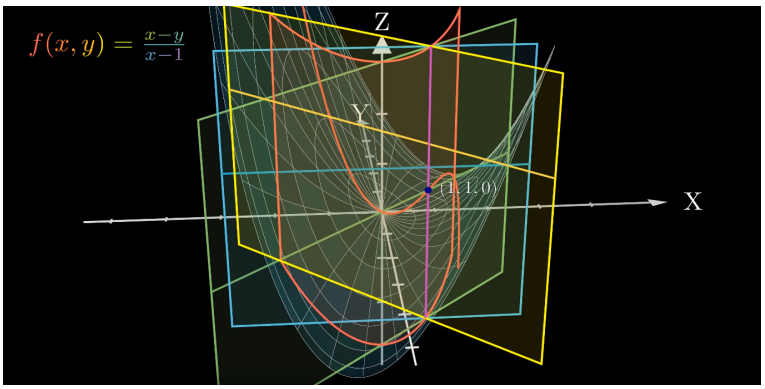
Throughout the lecture notes, the following topics have been covered:

- Introduction to Limits of the multivariable functions
- Epsilon-Delta definition of the limits of multivariable functions
- Methods for computing the limits of multivariable functions
- Introduction to Continuity of the multivariable functions

The following animation's screenshot gives a visulaisation on epsilon-delta definition of the limits of multivariable functions.



The following animation's screenshot shows the different paths of approach to a limit point.



GitHub: Multivariable Limits and Continuity

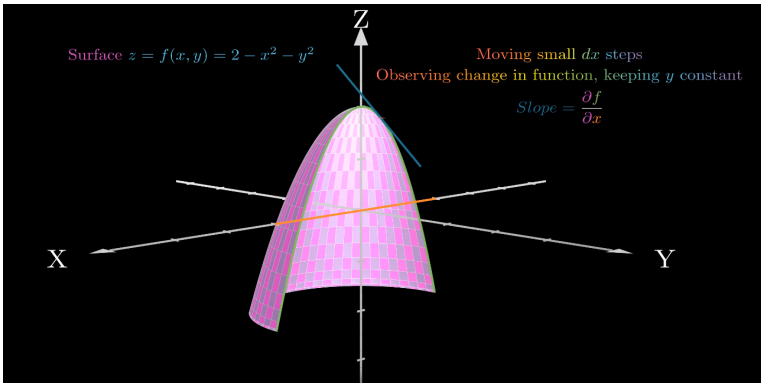
Partial Derivatives

Lecture Notes: Partial Derivatives

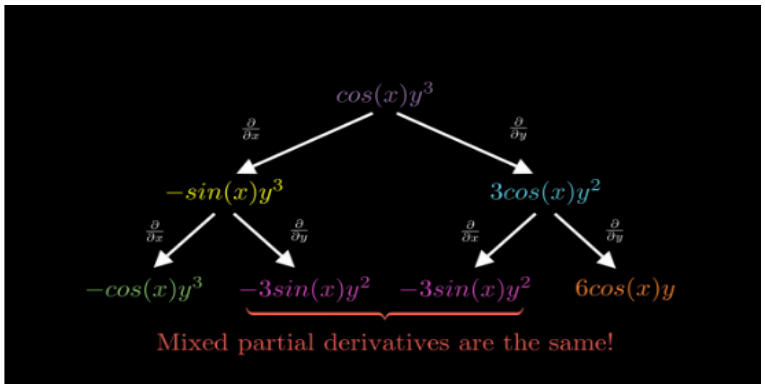
Throughout the lecture notes, the following topics have been covered:

- Introduction to partial derivatives
- Partial derivatives of multivariable functions
- Higher-order partial derivatives
- Clairaut's theorem
- Chain rule and it's application in the back propagation algorithm

The following animation's screenshot shows the partial derivatives along X axis.



The following animation's screenshot shows the proof of clairaut's rule.



GitHub: Partial Derivatives

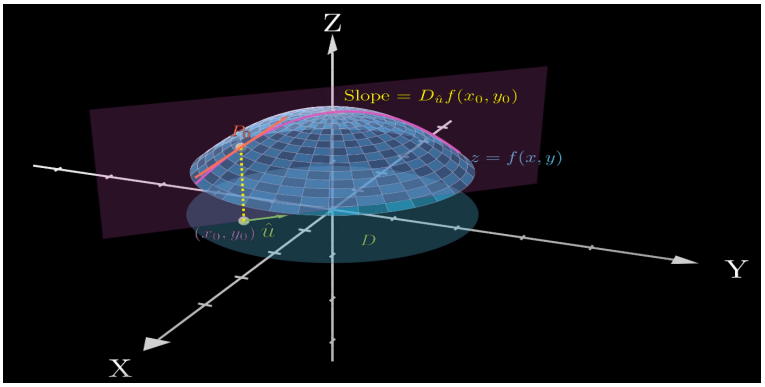
Directional Derivatives

Lecture Notes: Directional Derivatives

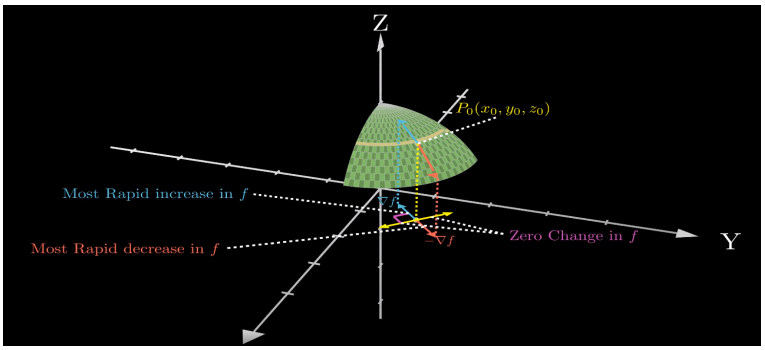
Throughout the lecture notes, the following topics have been covered:

- Introduction to directional derivatives
- Directional derivatives as the product of gradient and vector U
- Introduction to gradients
- Directional derivatives in terms of gradient
- Gradient is normal to level curves

The following animation's screenshot shows the geometrical significance of the directional derivative of a function of two variables.



The following animation's screenshot depicts that the gradient indicates the maximum and minimum values of the directional derivative at a point.



GitHub: Directional Derivatives

Conclusion

It has been a wonderful experience, understanding and visiting a few topics studied earlier and presenting it in a new bright way. My perception towards mathematics has changed, by understanding the very beauty of concept visualization.

This fellowship has given me a wonderful opportunity to visualise these beautiful mathematical concepts through animations and was rewarding in the development of my mathematical knowledge. Apart from this, I got an understanding on how to write lecture notes that are concise and easier to understand.

I will carry on the knowledge that I have gained from this fellowship ahead, and hope that I will have the opportunity to work again in the near future.