FOSSEE Summer Fellowship Report

On

Linear Transformations

Submitted by

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Acknowledgement

First of all, I would like to express my sincere and deepest gratitude to **FOSSEE IIT Bombay** for providing me this golden opportunity for learning and enhancing my knowledge in Linear Algebra.

I am deeply indebted to **Prof. Prabhu Ramachandran** for his time and valuable feedback. His suggestion helped me a lot in improving my lecture notes and animations.

The completion of my fellowship would not have been possible without the support and encouragement of my mentors **Mr. Purusharth Saxena** and **Ms. Sharanya Achut**.

Many thanks to Grant Sanderson for his contribution to the open-source python-based animation library, Manim. It is indeed a useful tool in making mathematical animations, in my opinion.

I very much appreciate the feedback and active responses to my queries by my fellows. I would also like to extend my sincere thanks to my parents, family members, and friends for supporting me throughout this fellowship.

Contents

Introduction

1.1 FOSSEE Animations and Summer Fellowship

1.1.1 What is FOSSEE?

FOSSEE (Free/Libre and Open Source Software for Education) project promotes the use of FLOSS tools in academia and research by encouraging students and faculty to use them in education and research. The FOSSEE project is part of the National Mission on Education through Information and Communication Technology (ICT), Ministry of Human Resource Development (MHRD), Government of India. The FOSSEE project promotes the use of FLOSS tools to improve the quality of education and research.

1.1.2 What is FOSSEE Animations?

The FOSSEE Animations Project works on making seemingly complex Science and Mathematics topics feel natural and approachable through animations. The students work with a mentor in creating a library of visualizations. FOSSEE Animations is a part of the FOSSEE project at the Indian Institute of Technology, Bombay.

A library of community-curated animations on Science and Math is maintained and improved. All the animations are made with open-source toolkits. It is freely available under a Creative Commons Attribution-ShareAlike 4.0 International License. Libraries like Manim, Mayavi, Blender, and VPython are employed for this purpose.

The animations themselves are open-source.You may find them here
(https://github.com/FOSSEE/FSF-mathematics-python-code-archive).

1.1.3 Summer Fellowship

Students across the country take part in summer fellowship to create animations on different mathematical topics. In summer fellowship, students selected work on mathematical topics such as Real Analysis, Calculus, and Linear Algebra. They submit lecture notes and animations to complement the notes.

In the fellowship, I worked on four topics of Linear Algebra. The animations created by me during this fellowship will help the students to understand and visualize topics of Linear Algebra. The animations were created using Manim, a Python-based animation library for creating explanatory mathematical videos.

Tools involved in creation of the lecture notes are:

- Python
- Manim
- LaTeX
- Git and GitHub
- ffmpeg
- MatplotLib

1.2 Topic for the Fellowship - Linear Transformations

Linear Algebra is a branch of mathematics that deals with linear equations and their representation in vector spaces using matrices. It is quite interesting to see how we can associate every matrice with a linear transformation. What interests me the most is that linear algebra is intensely is used in Machine Learning(an application of Artificial Intelligence).

GitHub: **https://github.com/FOSSEE/FSF-mathematics-python-code-archive/tree/master/FSF-2020/linear-algebra/linear-transformations**

Linear Transformations

Link for the subtopic : Linear Transformations (https://math.animations.fossee.in/contents/linear-algebra/linear-transformations/linear-transformations-(linear-maps))

Lecture notes cover the answer to the question - ”why are linear transformations linear” (visually explained using an animation). Notes also describe the two forms of Linear Transformations. Animations give the visual implications of Linear Transformations. This lecture notes provide a detailed description of common linear transformations such as:

- Scaling (Uniform and Non-Uniform)
- Shearing (Horizontal Shear and Vertical Shear)
- Rotation

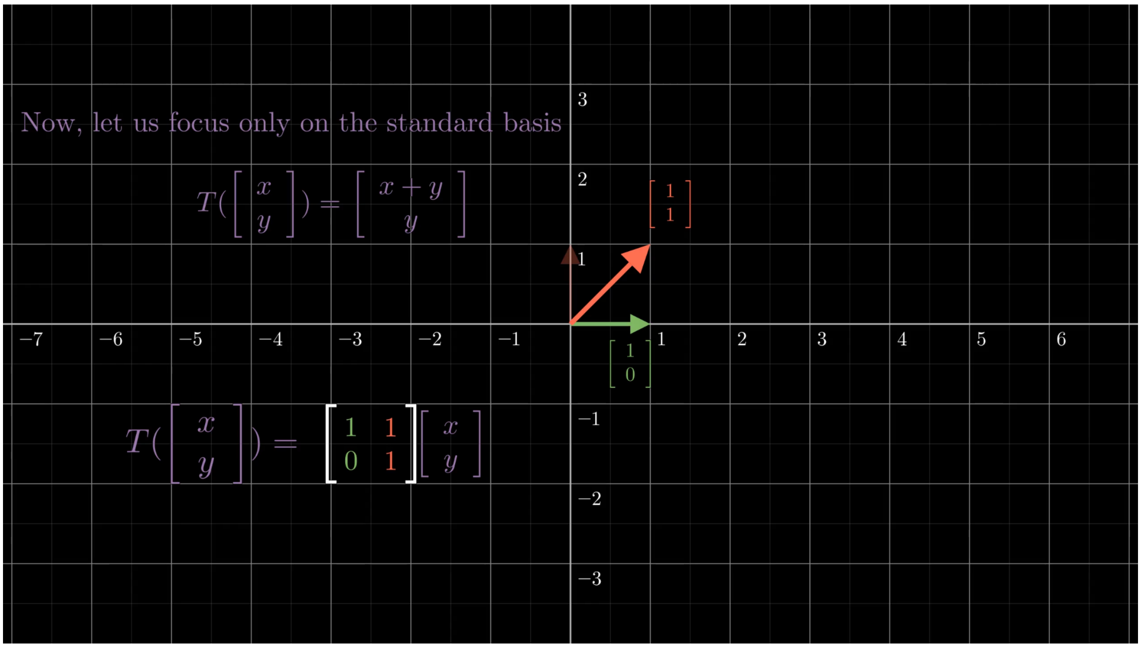
Now, let us focus only on the standard basis
$T(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} x+y \\ y \end{bmatrix}$
$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
$T(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$
−7 −6 −5 −4 −3 −2 −1 1 2 3 4 5 6
3 2 1 −1 −2 −3

Figure 2.1: Matrix Of Linear Transformation

GitHub: https://github.com/FOSSEE/FSF-mathematics-python-code-archive/tree/master/FSF-2020/linear-algebra/linear-transformations/Linear-Transformations-(Linear-Maps)

The Four Fundamental Subspaces

Link for the subtopic : The Four Fundamental Subspaces (https://math.animations.fossee.in/contents/linear-algebra/linear-transformations/the-four-fundamental-subspaces)

This lecture notes cover the method of finding the basis and hence dimension of all the four fundamental subspaces(Column Space, Null Space, Row Space, and Left Null Space) with examples. Animations included in notes takes care of the visual explanation of all the four fundamental subspaces.

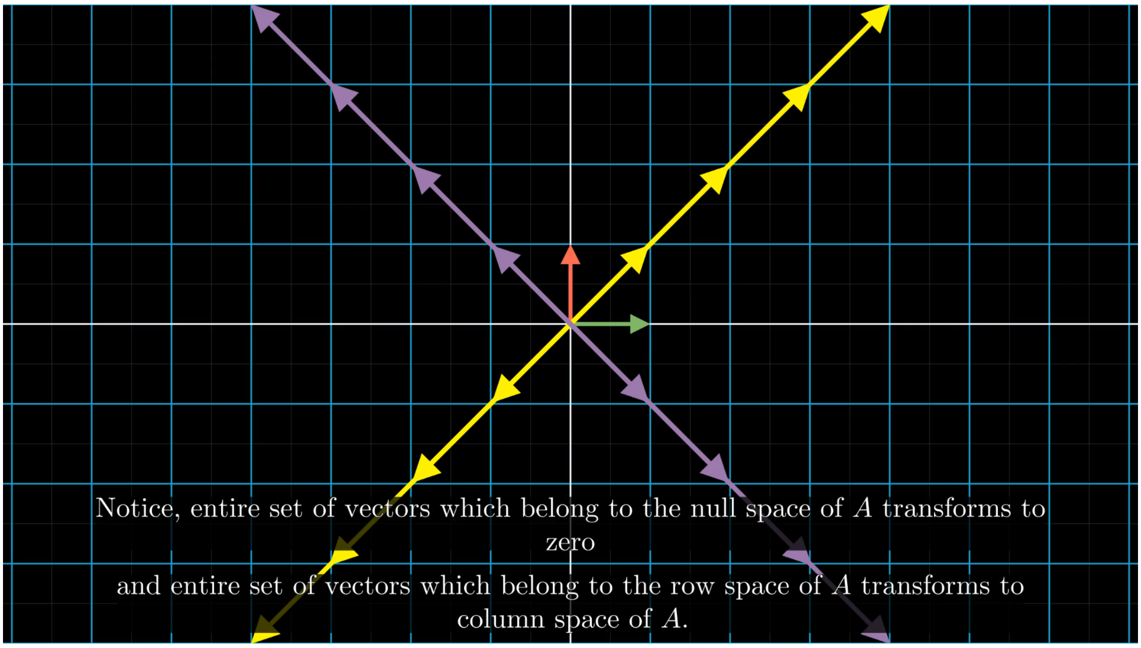
Notice, entire set of vectors which belong to the null space of A transforms to zero
and entire set of vectors which belong to the row space of A transforms to column space of A.

Figure 3.1: Relating the four fundamental subspaces.

GitHub: https://github.com/FOSSEE/FSF-mathematics-python-code-archive/tree/master/FSF-2020/linear-algebra/linear-transformations/The-Four-Fundamental-Subspaces

Orthonormal Bases

Link for the subtopic : Orthonormal Bases (https://math.animations.fossee.in/contents/linear-algebra/linear-transformations/orthonormal-bases))

These are definition oriented lecture notes. It includes an example based explanation of how projection property of orthonormal bases is helpful to minimize the calculation. Animation use example to verify the projection property of an orthonormal basis. The application section provides a good overview of how the orthonormal basis is helpful in other chapters of Linear Algebra.

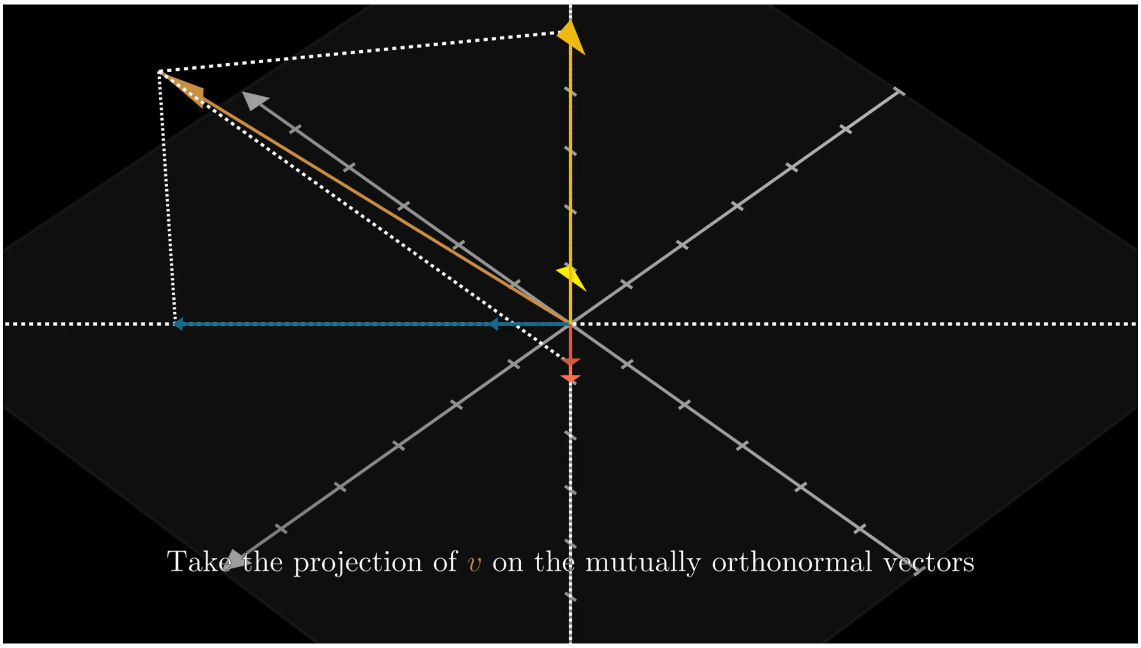
Take the projection of v on the mutually orthonormal vectors

Figure 4.1: Explaining how the projection property of orthonormal bases works

GitHub: https://github.com/FOSSEE/FSF-mathematics-python-code-archive/tree/master/FSF-2020/linear-algebra/linear-transformations/Orthonormal-Basis

Gram-Schmidt Orthogonalization Process

Link for the subtopic : Gram-Schmidt Orthogonalization Process (https://math.animations.fossee.in/contents/linear-algebra/linear-transformations/gram-schmidt-orthogonalization-process)

Lecture notes explain how we the Gram-Schmidt Orthogonalization Process and the projection property of orthonormal bases are connected. It also covers the algorithm explanation and its generalization. Animations try to show how the algorithm works. Video 3 is an animation which step by step guides you through the algorithm and gives a feel of how each step is contributing to the process. It provides us an overall visual procedure of how the Gram-Schmidt Orthogonalization Process will give orthonormal bases with the help of an example.

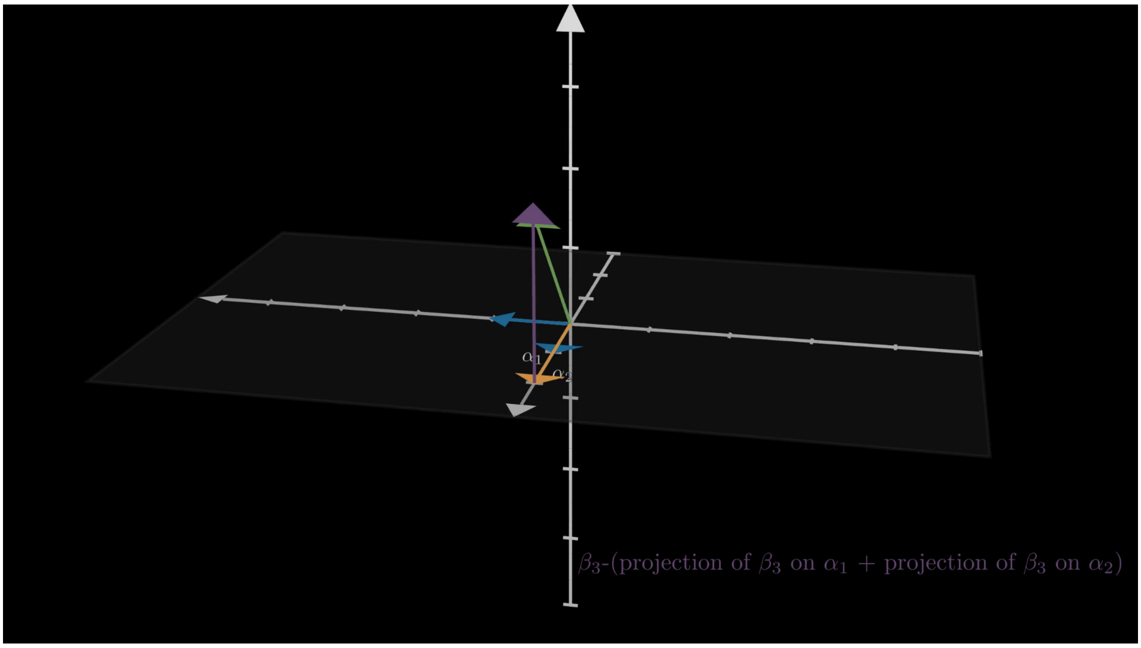
α_1
α_2
β_3-(projection of β_3 on α_1 + projection of β_3 on α_2)

Figure 5.1: Algorithm of Gram-Schmidt Orthogonalization Process

GitHub: https://github.com/FOSSEE/FSF-mathematics-python-code-archive/tree/master/FSF-2020/linear-algebra/linear-transformations/Gram-Schmidt-Orthonormalization-Process

Conclusion

I learned a lot about linear algebra, linear transformations in particular. I altogether got a new perspective of the linear transformations and the four fundamental subspaces. Also, this fellowship helped to understand the power of animations. I figured out that one can understand the concepts visually in a much easier way than the standard methodology of teaching. Mathematical concepts suddenly became a lot easier to understand and hard to forget. This fellowship helped me in developing the art of managing and maintain a proportion of work and play. I think after this fellowship, I am much more efficient in completing the tasks assigned. In the end, I would like to thank all the people who helped me in the construction of these lecture notes. My experience with FOSSEE was exceptionally helping in enhancing my mathematical knowledge. It was a very memorable experience.