CFD using OpenFOAM
Lecture 6: Essential Flow Governing Laws & OpenFOAM Implementation

Part II : Discretisation of Flow Governing Laws

Instructor : Sumant R Morab (Ph.D Research Scholar)
Co-ordinator : Prof. Janani S Murallidharan
Indian Institute of Technology, Bombay

Outline

Control Volume Notations

Mass Conservation

Momentum Conservation

Discretisation Scheme

Challenges and Solution

- Framework : Eulerian and Lagrangian

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- Conservation Equations in Fluid Dynamics

Recap : Governing Laws for Fluid Dynamics

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- Momentum Conservation (2D):

$$x\text{-Momentum: } \rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\partial p}{\partial x}$$

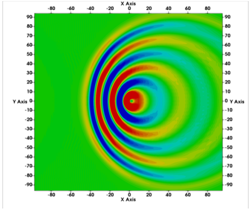
$$y\text{-Momentum: } \rho\frac{\partial v}{\partial t} + \rho u\frac{\partial v}{\partial x} + \rho v\frac{\partial v}{\partial y} = \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\partial p}{\partial y}$$

Division of contents for CFD & OpenFOAM Implementation

Part I : Governing Laws and PDE Format → $$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

Part II : Discretisation & Solution Methodology → $$\frac{U_{i,j+1} - U_{i,j-1}}{\Delta x} = 0$$

Part III : OpenFOAM Implementation & Illustration →



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Notations used for Finite Volume Discretisation

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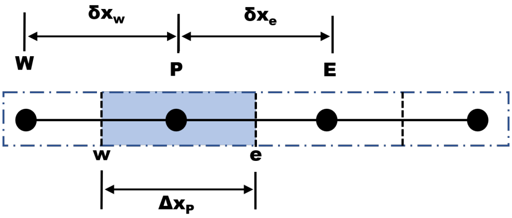

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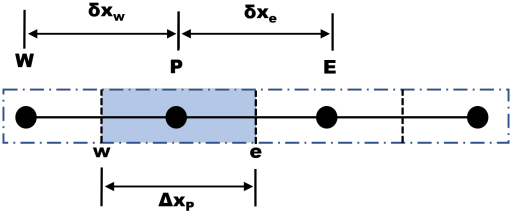

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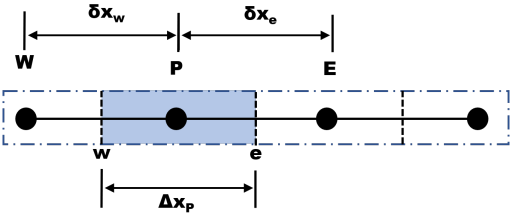

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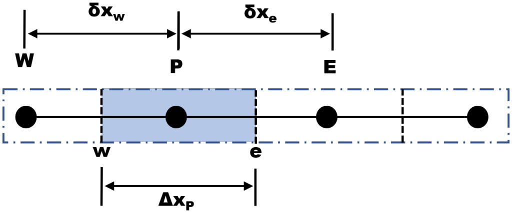

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$$\int\int_V\int(\vec{\nabla}\cdot\vec{F})dV = \int\int_A(\vec{F}\cdot\vec{n})dA \tag{1}$$

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$$\implies (\rho u_x)_e - (\rho u_x)_w = 0$$

Discretisation of X-Momentum Equation

- The governing equation for X-momentum conservation in 1D domain can be written as

$$\frac{\partial(\rho u_x)}{\partial t} + u_x \frac{\partial(\rho u_x)}{\partial x} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\mu \frac{\partial u_x}{\partial x}\right) \tag{3}$$

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$$\implies \frac{\rho\left(u_x^{t+\Delta t} - u_x^t\right)}{\Delta t} + (u_x \rho u_x)_e - (u_x \rho u_x)_w = -(p_e - p_w) + (\mu \frac{\partial u_x}{\partial x})_e - (\mu \frac{\partial u_x}{\partial x})_w$$

Properties of a Discretisation Scheme

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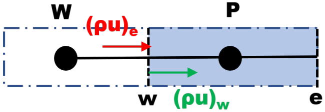
1. **Conservativeness**:
(flux out of a C.V) = (flux in of neighbouring C.V)



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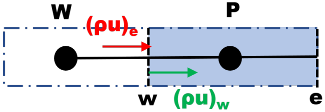
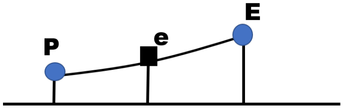


2. **Boundedness**:
values of variables @ face should be intermediate i.e, for upstream flow ; $\phi_W \leq \phi_w \leq \phi_P$



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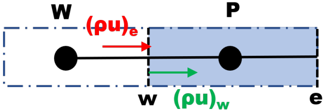

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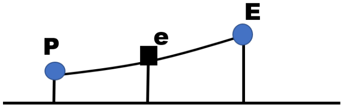
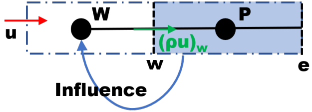


3. **Transportiveness**:
Calculation of variable value/gradient should consider effect of flow into account



1. Central Difference/Linear Interpolation : (Does not satisfy transportiveness)

$$\phi_e = \frac{\Delta V_P \phi_E + \Delta V_E \phi_P}{\Delta V_P + \Delta V_E} ; \phi_w = \frac{\Delta V_P \phi_W + \Delta V_W \phi_P}{\Delta V_P + \Delta V_W} \tag{4}$$

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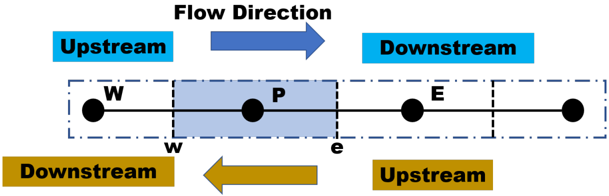

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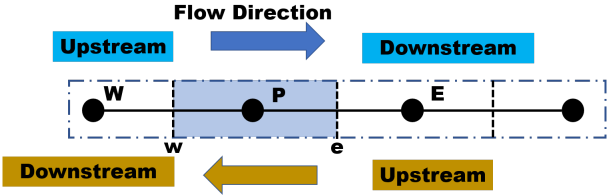

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$$\phi_f = \phi_U \quad (5)$$

Common Discretisation Schemes for Advection

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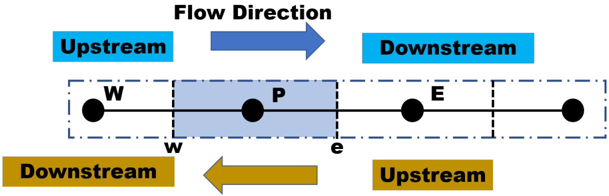

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3. Second Order Upwind (SOU) :

$$\phi_f = 1.5\phi_U - 0.5\phi_{UU} \quad (6)$$

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- **Non-Linearity of Advection Term:**
  Advection term consists of $u_x(\rho u_x)$ term, the equation becomes non-linear
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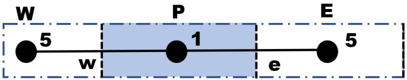

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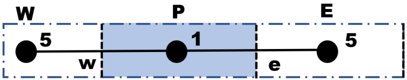

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$$\frac{\partial p}{\partial x} = (p_e - p_w)/\Delta x_P$$

$$\frac{\partial p}{\partial x} = \frac{((P_E + P_P) - (P_W + P_P))}{(2\Delta x_P)} = 0$$

In order to address the issue of non-availability of specific pressure equation in conservation laws, a predictor corrector approach named as '**Semi-Implicit Method for Pressure Linked Equations**' (SIMPLE) is employed as given below

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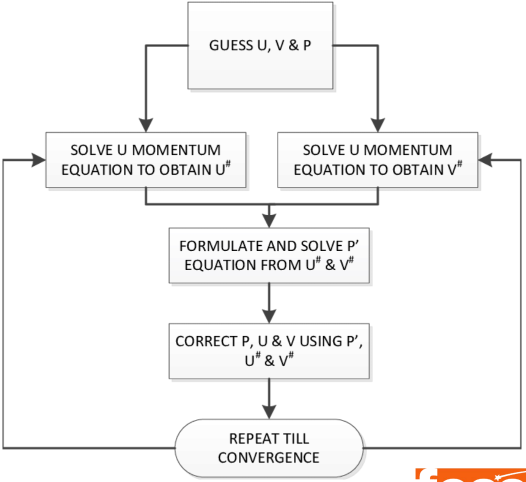

Staggered Grid Arrangement

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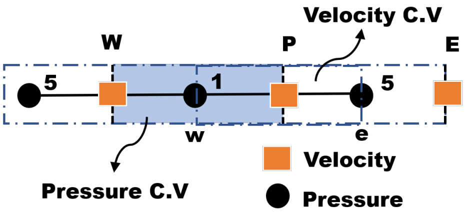

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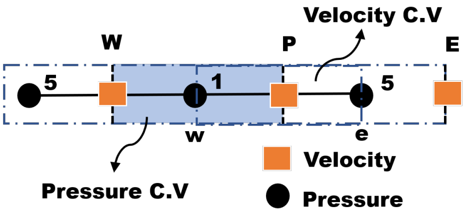

$$\frac{\partial p}{\partial x} = (p_e - p_w)/\Delta x_P$$

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2. Discretisation Procedure for Mass Conservation

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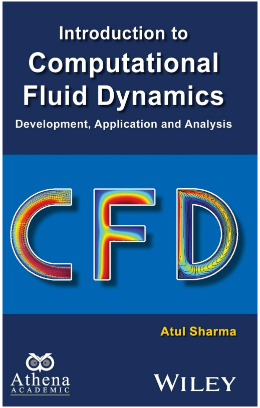
In the next lecture, we shall look into the implementation details in OpenFOAM code and see a sample example.

References

1. Sharma, A. (2016). Introduction to computational fluid dynamics: development, application and analysis. John Wiley & Sons.
2. NPTEL AE 20 Course (https://nptel.ac.in/courses/101105085)
3. https://www.openfoam.com/



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