CFD using OpenFOAM
Lecture 5: Essential Flow Governing Laws & OpenFOAM Implementation

Part I : Physical Flow Governing Laws

Instructor : Sumant R Morab (Ph.D Research Scholar)
Co-ordinator : Prof. Janani S Murallidharan
Indian Institute of Technology, Bombay

Outline

- Governing Equation :

$$\left\{\frac{\partial(\rho C_p T)}{\partial t}\right\} + \left(\frac{\partial(\rho u_x C_p T)}{\partial x} + \frac{\partial(\rho u_y C_p T)}{\partial y}\right) = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] \tag{1}$$

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- Solution Methodology : Iterative scheme with implicit time-stepping
- Illustrative Problem : (1) Unsteady 2D heat conduction in a metallic slab and (2) Heat convection in channel with backward-facing step

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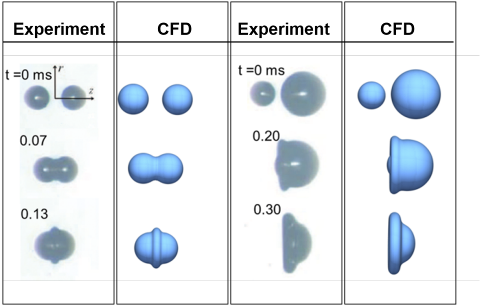

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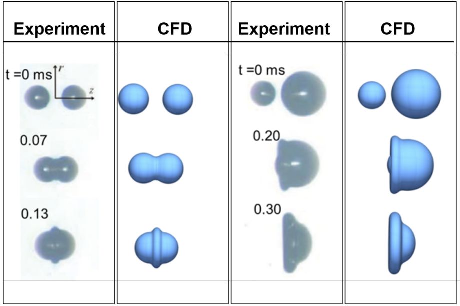

- Physical reality achieved by solving correct physical model(s).

Part I : Governing Laws and PDE Format →

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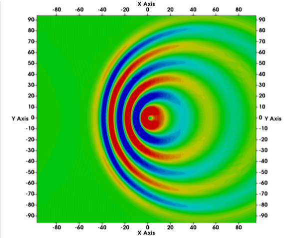
Part III : OpenFOAM Implementation & Illustration



Framework for Governing Laws

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  (1) Mass Conservation (Continuity)
  (2) Momentum Conservation (Newton's 2^{nd} Law)

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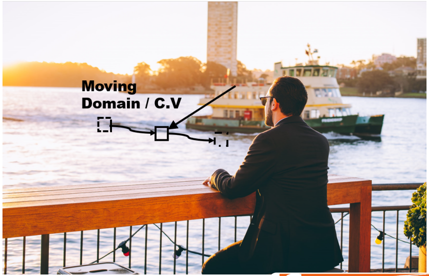
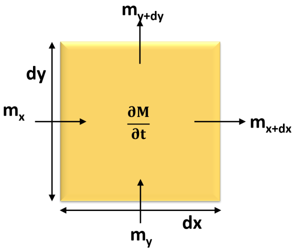
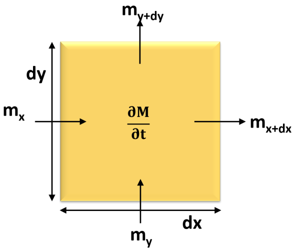
Eulerian Framework

Lagrangian Framework

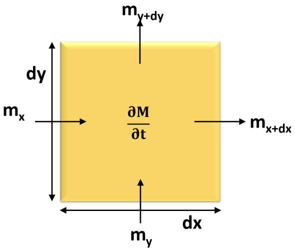



m_{y+dy}
dy
m_x
$\frac{\partial M}{\partial t}$
m_{x+dx}
dx
m_y

Mass Conservation



- Rate of increase of mass inside a C.V = Net rate of Mass influx to C.V - Net rate of Mass outflux from C.V. i.e,



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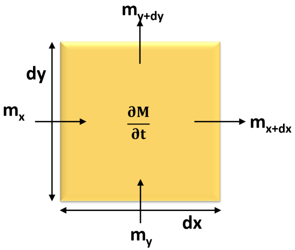
$$\frac{\partial M_{vol.}}{\partial t} = \frac{m_x - m_{x+dx}}{dx} + \frac{m_y - m_{y+dy}}{dy} \qquad (2)$$

Where $M_{vol.} = M/Vol. = \rho$, $m_x = \rho u$ and $m_y = \rho v$



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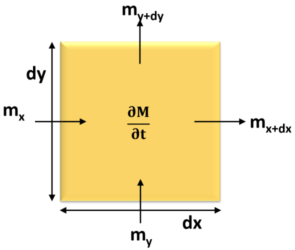

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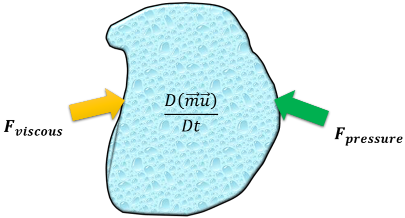
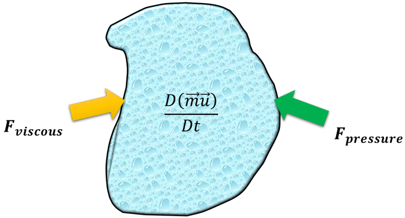
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

Momentum Conservation - General Principle





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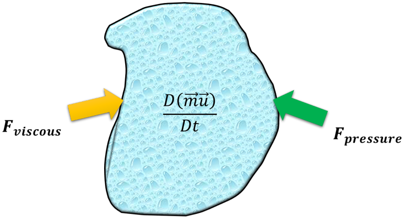

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1. $D()/Dt$ = Rate of change of momentum
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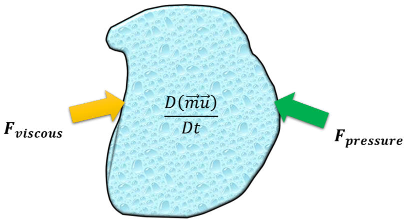

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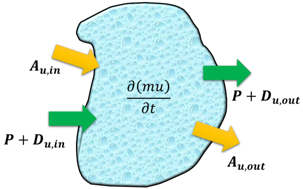
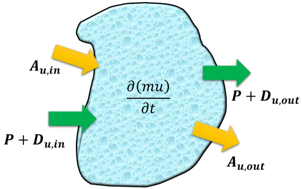
2. $F_{viscous} \rightarrow$ Force due to shear between two layers of fluid, $F_{pressure} \rightarrow$ Hydrodynamic pressure

Momentum Conservation - X Direction





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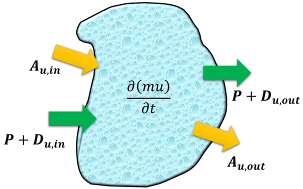

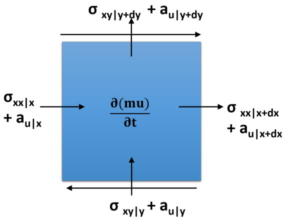

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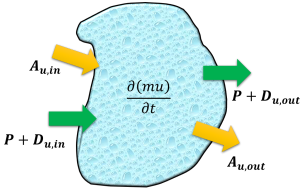

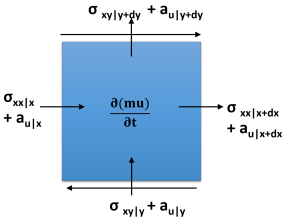

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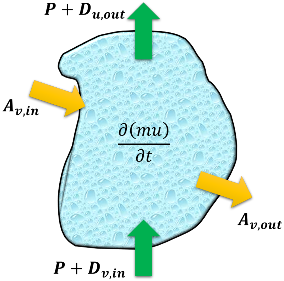
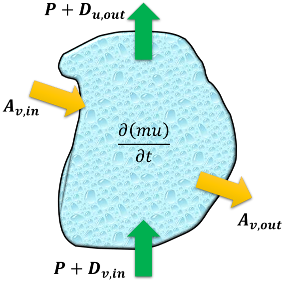
Momentum Conservation - Y Direction





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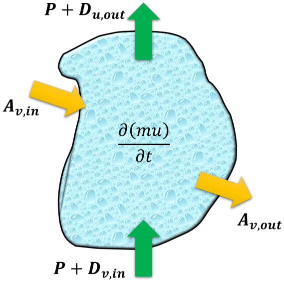

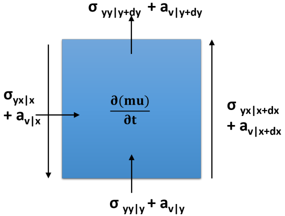

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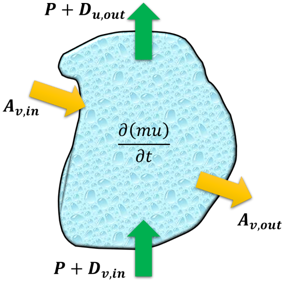

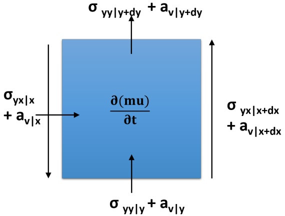

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- How to convert above laws in terms of fluid velocity & pressure ?
- Similar to Fourier's law of heat conduction, Fluid Dynamics involves use of Newton's stress strain relation to serve the purpose.

$$\sigma_{xx} = -p + 2\mu\dot{\epsilon}_{xx} \tag{8}$$

$$\sigma_{yy} = -p + 2\mu\dot{\epsilon}_{yy} \tag{9}$$

$$\sigma_{xy} = \sigma_{yx} = 2\mu\dot{\epsilon}_{xy} \tag{10}$$

where 'μ' indicates dynamic viscosity, $\dot{\epsilon}$ indicates strain rate which can be expressed in terms of fluid velocities as follows

$$\dot{\epsilon}_{xx} = \mu\frac{\partial u}{\partial x};\ \dot{\epsilon}_{yy} = \mu\frac{\partial v}{\partial y};\ and\ \dot{\epsilon}_{xy} = \frac{\mu}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \tag{11}$$

Final Governing Equations

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y\text{-Momentum: } \rho\frac{\partial v}{\partial t} + \rho u\frac{\partial v}{\partial x} + \rho v\frac{\partial v}{\partial y} &= \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\partial p}{\partial y}
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$$

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2. Framework for Governing Laws (Eulerian & Larangian)

Summary

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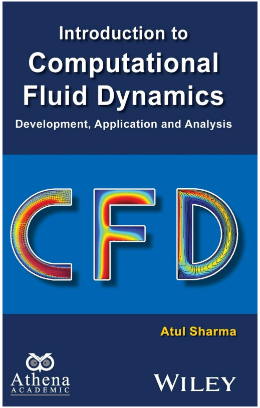
In the next lecture, we shall look into the discretised form and some of the challenges involved in solving these equations.

References

1. Sharma, A. (2016). Introduction to computational fluid dynamics: development, application and analysis. John Wiley & Sons.
2. Goyal, N., Shaikh, J., & Sharma, A. (2020). Bubble entrapment during head-on binary collision with large deformation of unequal-sized tetradecane droplets. Physics of Fluids, 32(12), 122114.
3. https://www.openfoam.com/



Thank you for listening!

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